MINITAB Manual For

David Moore and George McCabe's

Introduction To The Practice of Statistics

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Contents

Preface

This Minitab manual is to be used as an accompaniment to *Introduction to the Practice of Statistics,* Fourth Edition, by David S. Moore and George P. McCabe, and to the CD-ROM that accompanies this text. We abbreviate the textbook title as IPS.

Minitab is a statistical software package that was designed especially for the teaching of introductory statistics courses. It is our view that an easy-to-use statistical software package is a vital and significant component of such a course. This permits the student to focus on statistical concepts and thinking rather than computations or the learning of a statistical package. The main aim of any introductory statistics course should always be the "why" of statistics rather than technical details that do little to stimulate the majority of students or, in our opinion, do little to reinforce the key concepts. IPS succeeds admirably in communicating the important basic foundations of statistical thinking, and it is hoped that this manual serves as a useful adjunct to the text.

It is natural to ask why Minitab is advocated for the course. In the author's experience, ease of learning and use are the salient features of the package, with obvious benefits to the student and to the instructor, who can relegate many details to the software. While more sophisticated packages are necessary for higher-level professional work, it is our experience that attempting to teach one of these in a course forces too much attention on technical aspects. The time students need to spend to learn Minitab is relatively small and that it is a great virtue. Further Minitab will serve as a perfectly adequate tool for many of the statistical problems students will encounter in their undergraduate education.

This manual is divided into two parts. Part I is an introduction that provides the necessary details to start using Minitab and in particular how to use worksheets. Not all the material in Part I needs to be absorbed on first reading. We recommend reading I.1–I.10 before starting to use Minitab. The material in I.11 is more for reference and for later reading. References are made to these sections later in the manual and can provide the stimulus to read them. Overall, the introductory Part I also serves as a reference for most of the nonstatistical commands in Minitab.

Part II follows the structure of the textbook. Each chapter is titled and numbered as in IPS. The last two chapters are not in IPS but correspond to optional material included on the CD-ROM. The Minitab commands relevant to doing the problems in each IPS chapter are introduced and their use illustrated. Each chapter concludes with a set of exercises, some of which are modifications of or related to problems in IPS and many of which are new and specifically designed to ensure that the relevant Minitab material has been understood. There are also appendices dealing with some more advanced features of Minitab, such as programming in Minitab and matrix algebra.

Minitab is available in a variety of versions and for different types of computing systems. In writing the manual, we have used Version 13 for Windows, as discussed in the references in Appendix F, but have tried to make the contents of the manual compatible with earlier versions and for versions running under other operating systems. The core of the manual is a discussion of the menu commands while not neglecting to refer to the session commands. Overall, we feel that the manual can be successfully used with most versions of Minitab.

This manual does not attempt a complete coverage of Minitab. Rather, we introduce and discuss those concepts in Minitab that we feel are most relevant for a student studying introductory statistics with IPS. We do introduce some concepts that are, strictly speaking, not necessary for solving the problems in IPS where we feel that they were likely to prove useful in a large number of data analysis problems encountered outside the classroom. While the manual's primary goal is to teach Minitab, generally we want to help develop strong data analytic skills in conjunction with the text and the CD-ROM.

Thanks to Patrick Farace and Chris Spavins of W. H. Freeman and Company for their help and consideration. Also thanks to Rosemary and Heather.

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URL: http://www.minitab.com

Part I

Minitab for Data Management

New Minitab commands discussed in this part

Calc I Calculator
Calc I Make Patterned Data
Edit I Copy Cells
Edit I Paste Cells
Edit I Undo Cut
Editor I Enable Command Language
Editor I Insert Columns
Editor I Make Output Editable
File I Exit
File I Other Files I Export Special Text
File I Other Files I Import Special Text
File I Print Worksheet
File I Save Current Worksheet As
Help
Manip I Code
Manip I Copy Columns
Manip I Erase Variables
Manip I Sort
Manip I Unstack
Window I Project Manager

Calc I Column Statistics
Calc I Row Statistics
Edit I Cut Cells
Edit I Select All Cells
Edit I Undo Paste
Editor I Insert Cells
Editor I Insert Rows
File I New
File I Open Worksheet
File I Print Session Window
File I Save Current Worksheet
File I Save Session Window As
Manip I Concatenate
Manip I Display Data
Manip I Rank
Manip I Stack

1 Manual Overview and Conventions

The manual is divided into two parts. Part I is concerned with getting data into and out of Minitab and giving you the tools necessary to perform various elementary operations on the data so that it is in a form in which you can carry out a statistical analysis. You do not need to understand everything in Part I to begin doing the problems in your course. Part II is concerned with the statistical analysis of the data set and the Minitab commands to do this. The chapters in Part II follow the chapters in *Introduction to the Practice of Statistics,* Fourth Edition, by David S. Moore and George P. McCabe, and to the CD-ROM that accompanies this text (IPS hereafter) and are numbered accordingly. Before you start on Chapter II.1, however, you should read I.1–I.10 and leave I.11 for later reading.

Minitab is a software package that runs on a variety of different types of computers and comes in a number of versions. This manual does not try to describe all the possible implementations or the full extent of the package. We limit our discussion to those features common to the most recent versions of Minitab and, in particular, Versions 12 and 13. Also, we present only those aspects of Minitab relevant to carrying out the statistical analyses discussed in IPS. Of course, this is a fairly wide range of analyses, but the full power of Minitab is not necessary. Depending on the version of Minitab you are using, there may be many more useful features, and we encourage you to learn and use them. Throughout the manual, we point out what some of the additional useful features of Minitab are and how you can go about learning how to use them. Version 13 refers to the most current version of Minitab at the time of writing this manual.

In this manual, special statistical or Minitab concepts will be highlighted in *italic* font. You should be sure that you understand these concepts. We will provide a brief explanation for any terms not defined in IPS. When a reference is made to a Minitab *session command* or *subcommand*, its name will be in **bold** font. Primarily, we will be discussing the *menu commands* that are available in Minitab. Menu commands are accessed by clicking the left button of the mouse on items in lists. We use a special notation for menu commands. For example,

A **I** B **I** C

is to be interpreted as left click the command A on the menu bar, then in the list that drops down, left click the command B, and, finally, left click C. The menu commands will be denoted in ordinary font (the actual appearance may vary slightly depending on the version of Windows you use). Any commands that we type and the output obtained will be denoted in `typewriter` font, as will the names of any files used by Minitab, variables, constants, and worksheets.

At the end of each chapter, we provide a few exercises that can be used to make sure you have understood the material. We recommend, however, that whenever possible you use Minitab to do the problems in IPS. While many problems can be done by hand, you will save a considerable amount of time and avoid errors by learning to use Minitab effectively. We also recommend that you try out the Minitab commands as you read about them, as this will ensure full understanding.

2 Accessing and Exiting Minitab

The first thing you should do is find out how to access the Minitab package for your course. This information will come from your instructor, system personnel, or from your software documentation if you have purchased Minitab to run on your own computer.

In some cases, this may mean you type a command such as **minitab** at a computer system prompt and then hit the Enter or Return key on the keyboard after you have logged on, i.e., provided a login name and password to the computer system being used in your course. Typically, you will see the prompt

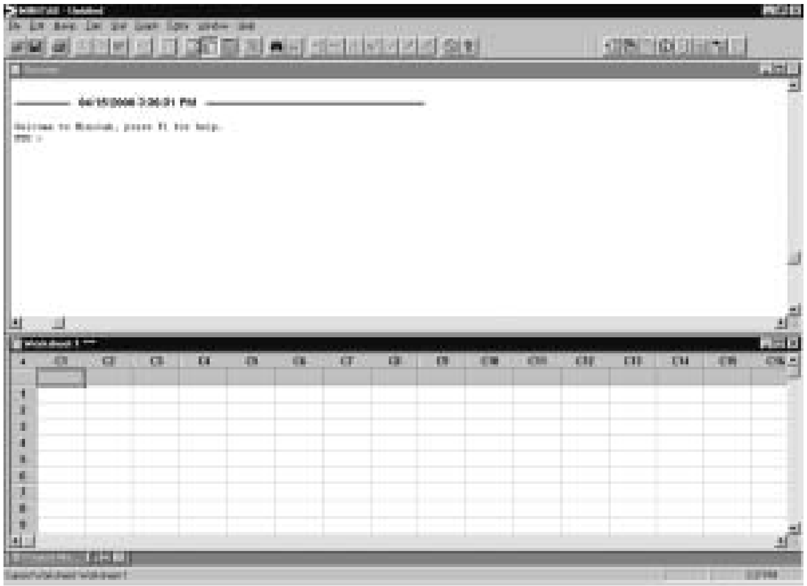
```
MTB >
```

on your screen, and this indicates that you have started a Minitab *session.*

In most cases, you will double click an icon, such as that shown in Display I.1, that corresponds to the Minitab program.

Display I.1: Minitab icon.

Alternatively, you can use the Start button and click on Minitab in the Programs list. In this case, the program opens with a *Minitab window,* such as the one shown in Display I.2. The Minitab window is divided into two sub-windows with the upper window called the *Session window* and the lower one called the *Data window.*

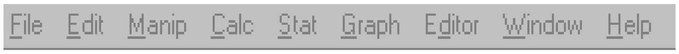


```
———— 04/15/2006 3:36:01 PM ————————————

Welcome to Minitab, press F1 for help.
MTB >
```

Display I.2: Minitab window.

Left clicking the mouse anywhere on a particular window brings that window to the foreground, i.e., makes it the *active* window, and the border at the top of the window turns dark blue. For example, clicking in the Session window will make the window containing the MTB > prompt active. Alternatively, you can use the command Window I Session in the *menu bar* at the top of the Minitab window to make this window active. You may not see the `MTB >` prompt in your Session window, and for this manual it is important that you do so. You can ensure that this prompt always appears in your Session window by using Edit I Preferences, doubleclick on Session Window in the Preferences list that comes up, clicking on the Enable radio button under Command Language in the Session Window Preferences, clicking on OK, and clicking on Save. Without the `MTB >` prompt, you cannot type commands to be executed in the Session window.

In the session window, Minitab *commands* are typed after the `MTB >` prompt and executed when you hit the Enter or Return key. For example, the first command you should learn is **exit,** as this takes you out of your Minitab session and returns you to the system prompt or operating system. Otherwise, you can access commands using the menu bar (Display I.3) that resides at the top of the Minitab window. For example, you can access the **exit** command using File I Exit. In many circumstances, using the menu commands to do your analyses is easy and convenient, although there are certain circumstances where typing the session commands is necessary. You can also exit by clicking on the × symbol in the upper right-hand corner of the Minitab window. When you exit, you are prompted by Minitab in a dialog window with the question, "Save changes to this Project before closing?" You can safely answer no to this question unless you are in fact using the Projects feature in Minitab as described in Appendix A. In I.8, we will discuss how to save the contents of a Data window before exiting. This is something you will commonly want to do.



Display I.3: Menu bar.

Immediately below the menu bar in the Minitab window is the *taskbar*. The taskbar consists of various icons that provide a shortcut method for carrying out various operations by clicking on them. These operations can be identified by holding the cursor over each in turn, and it is a good idea to familiarize yourself with these. Of particular importance are the Cut Cells, Copy Cells, and Paste Cells icons, which are available when a Data window is active. When the operation associated with an icon is not available the icon is faded.

Minitab is an interactive program. By this we mean that you supply Minitab with input data, or tell it where your input data is, and then Minitab responds instantaneously to any commands you give telling it to do something with that data. You are then ready to give another command. It is also possible to run a collection of Minitab commands in a batch program; i.e., several Minitab commands are executed sequentially before the output is returned to the user. The batch version is useful when there is an extensive number of computations to be carried out. You are referred to Appendix C for more discussion of the batch version.

3 Files Used by Minitab

Minitab can accept input from a variety of files and write output to a variety of files. Each file is distinguished by a *file name* and an *extension* that indicates the type of file it is. For example, `marks.mtw` is the name of a file that would be referred to as ‘marks’ (note the single quotes around the file name) within Minitab. The extension `.mtw` indicates that this is a Minitab worksheet. We describe what a worksheet is in I.5. This file is stored somewhere on the hard drive of a computer as a file called `marks.mtw.`

There are other files that you will want to access from outside Minitab, perhaps to print them out on a printer. Depending on the version of Minitab you are using, to do this, you may have to exit Minitab and give the relevant system print command together with the full path name of the file you wish to print. As various implementations of Minitab differ as to where these files are stored on the hard drive, you will have to determine this information from your instructor or documentation or systems person. For example, in the windows environment the full path name of the file could be

```
c:\Program Files\MTBWIN\Data\marks.mtw
```

or something similar. This path name indicates that the file `marks.mtw` is stored on the C hard drive in the directory called `Program Files\Mtbwin\Data.` We will discuss several different types of files in this chapter.

In many versions of Minitab, there are restrictions on file names. For example, in earlier versions a file name can be at most eight characters in length using any symbols except # and ’ and the first character cannot be a blank. There is no length restriction on file names in Versions 12 or 13. It is generally best to name your files so that the file name reflects its contents. For example, the file name `marks` may refer to a data set composed of student marks in a number of courses.

4 Getting Help

At times, you may want more information about a command or some other aspect of Minitab than this manual provides, or you may wish to remind yourself of some detail that you have partially forgotten. Minitab contains an online manual that is very convenient. You can access this information directly by clicking on Help in the Menu bar and using the table of _Contents or doing a Search of the manual for a particular concept.

From the `MTB >` prompt, you can use the **help** command for this purpose. Typing **help** followed by the name of the command of interest and hitting Enter will cause Minitab to produce relevant output. For example, asking for help on the command **help** itself via the command

```
MTB >help help
```

will give you an overview of what help information can be accessed on your system. The **help** command should be used to find out about session commands.

5 The Worksheet

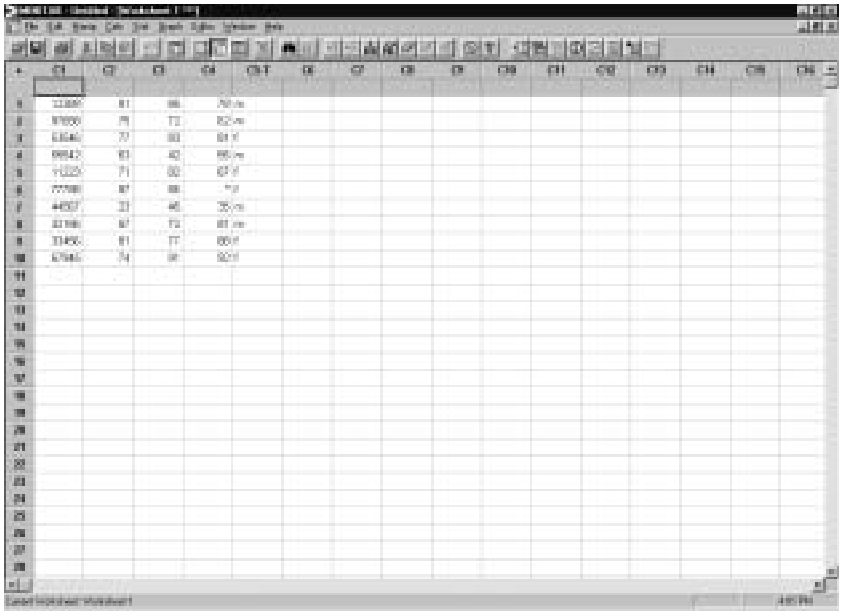
The basic structural component of Minitab is the *worksheet*. Basically, the worksheet can be thought of as a big rectangular array, or matrix, of *cells* organized into rows and columns as in the Data window of Display I.2. Each cell holds one piece of data. This piece of data could be a number, i.e. *numeric data*, or it could be a sequence of characters, such as a word or an arbitrary sequence of letters and numbers, i.e., *text data*. Data often comes as numbers, such as 1.7, 2.3, . . . but sometimes it comes in the form of a sequence of characters, such as black, brown, red, etc. Typically, sequences of characters are used as identifiers in classifications for some variable of interest, e.g., color, gender. A piece of text data can be up to 80 characters in length in Minitab. Version 13 also allows for *date data*, which is data especially formatted to indicate a date, for example, 3/4/97. We will not discuss date data.

If possible, try to avoid using text data with Minitab, i.e., make sure all the values of a variable are numbers, as dealing with text data in Minitab is more difficult. For example, denote colors by numbers rather than by names. Still there will be applications where data comes to you as text data, e.g., in a computer file, and it is too extensive to convert to numeric data. So we will discuss how to input text data into a Minitab worksheet, but we recommend that in such cases you convert this to numeric data, using the methods of I.11.3, once it has been input. In Version 13 of Minitab it is somewhat easier to deal with text data than earlier versions, and this proviso is not as necessary.

Display I.4 provides an example of a worksheet. Notice that the columns are labeled C1, C2, etc. and the rows are labeled 1, 2, 3, etc. We will refer to the worksheet depicted in Display I.4 as the `marks` worksheet hereafter and will use it throughout Part I to illustrate various Minitab commands and operations.

Data arises from the process of taking measurements of variables in some real-world context. For example, in a population of students, suppose that we are conducting a study of academic performance in a Statistics course. Specifically, suppose that we want to examine the relationship between grades in Statistics, grades in a Calculus course, grades in a Physics course and gender. So we collect the following information for each student in the study: student number, grade in Statistics, grade in Calculus, grade in Physics, and gender. Therefore, we have 5 variables – student number and the grades in the three subjects are *numeric variables*, and gender is a *text variable*. Let us further suppose that there are 10 students in the study.

Display I.4 gives a possible outcome from collecting the data in such a study. Column C1 contains the student number (note that this is a categorical variable even though it is a number). The student number primarily serves as an identifier so that we can check that the data has been entered correctly. This is something you should always do as a first step in your analysis. Columns C2–C4 contain the student grades in their Statistics, Calculus, and Physics courses and column C5 contains the gender data. Notice that a column contains the values collected for a single variable, and a row contains the values of all the variables for a single student. Sometimes, a row is referred to as an *observation* or *case*. Observe that the data for this study occupies a 10×5 subtable of the full worksheet. All of the other blank entries of the worksheet can be ignored, as they are undefined.



↓	C1	C2	C3	C4	C5-T
1	[illegible]	81	85	78	m
2	[illegible]	75	72	62	m
3	[illegible]	77	83	81	f
4	[illegible]	63	42	55	m
5	[illegible]	71	82	67	f
6	[illegible]	87	88	[illegible]	f
7	[illegible]	23	45	35	m
8	[illegible]	67	73	81	m
9	[illegible]	81	77	88	f
10	[illegible]	74	91	92	f

Display I.4: The marks worksheet.

There will be limitations on the number of columns and rows you can have in your worksheet, and this depends on the particular implementation of Minitab you are using. So if you plan to use Minitab for a large problem, you should check with the system person or further documentation to see what these are. For example, in some versions of Minitab there is a limitation of 5000 cells. So there can be one variable with 5000 values in it, or 50 variables with 100 values each, etc.

Associated with a worksheet is a table of *constants*. Typically, these are numbers that you want to use in some arithmetical operation applied to every value in a column. For example, you may have recorded heights of people in inches and want to convert these to heights in centimeters. You must multiply every height by the value 2.54. The Minitab constants are labeled K1, K2, etc. Again, there are limitations on the number of constants you can associate with a worksheet. For example, in many versions there can be at most 1000 constants. So to continue with the above problem, we might assign the value 2.54 to K1. In I.7.4, we show how to make such an assignment, and in I.10.1 we show how to multiply every entry in a column by this value.

In Version 13 of Minitab, there is an additional structure beyond the worksheet called the *project*. A project can have multiple worksheets associated with it. Also, a project can have associated with it various graphs and records of the commands you have typed and the output obtained while working on the worksheets. Projects, which are discussed in Appendix A, can be saved and retrieved for later work. Projects .

6 Minitab Commands

We will now begin to introduce various Minitab commands to get data into a worksheet, edit a worksheet, perform various operations on the elements of a worksheet, and save and access a saved worksheet. Before we do, however, it is useful to know something about the basic structure of all Minitab commands. Associated with every command is of course its *name,* as in File I Exit and Help. Most commands also take *arguments,* and these arguments are column names, constants, and sometimes file names.

Commands can be accessed by making use of the File, Edit, Manip, Calc, Stat, Graph and Editor entries in the menu bar. Clicking any of these brings up a list of commands that you can use to operate on your worksheet. The lists that appear may depend on which window is active, e.g., either a Data window or the Session window. Unless otherwise specified, we will always assume that the Session window is active when discussing menu commands. If a command name in a list is faded, then it is not available.

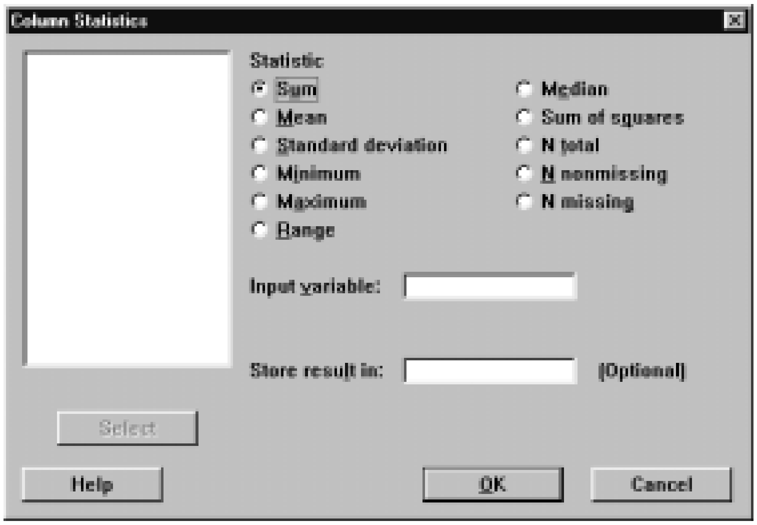
Typically, using a command from the menu bar requires the use of a *dialog box* or *dialog window* that opens when you click on a command in the list. These are used to provide the arguments and subcommands to the command and specify where the output is to go. Dialog boxes have various boxes that must be filled in to correctly execute a command. Clicking in a box that needs to be filled in typically causes a *variable list* to appear in the left-most box, of all items in the active worksheet that can be placed in that box. Double clicking on items in the variable list places them in the box, or, alternatively, you can type them in directly. When you have filled in the dialog box and clicked OK, the command is printed in the Session window and executed. Any output is also printed in the Session window. Dialog boxes have a Help button that can be used to learn how to make the entries.

For example, suppose that we want to calculate the *mean* of column C2 in the worksheet marks. Then the command Calc I Column Statistics brings up the dialog box shown in Display I.5. Notice that the radio button Sum is filled in. Clicking the radio button labelled Mean results in this button being filled in and the Sum button becoming empty. Whichever button is filled in will result in that statistic being calculated for the relevant columns when we finally implement the command by clicking OK.

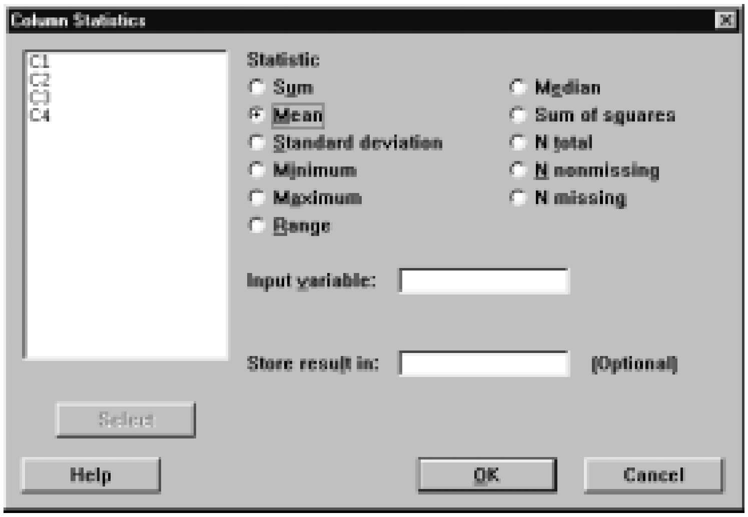
Currently, there are no columns selected, but clicking in the Input variable box brings up a list of possible columns in the display window on the left. The results of these operations are shown in Display I.6. We double click on C2 in the variable list, which places this entry in the Input variable box as shown in Display I.7. Alternatively, we could have simply typed this entry into the box. After clicking the OK button, we obtain the output

```
Mean of C2 = 69.900
```

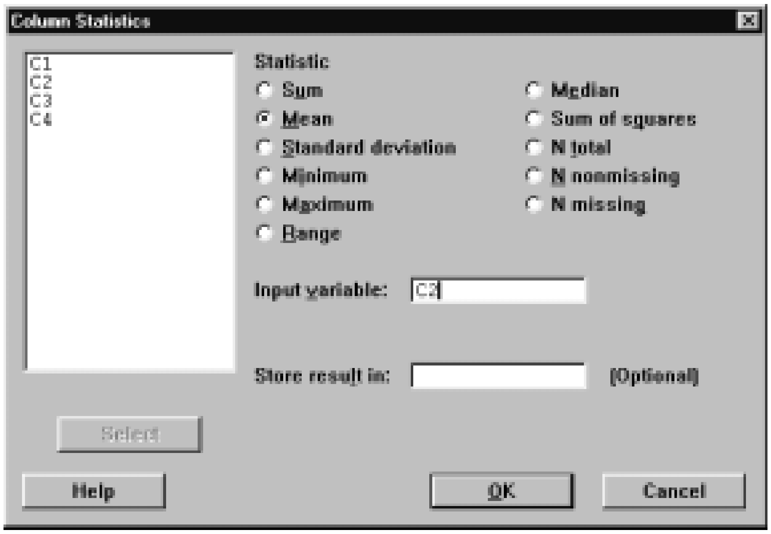
in the Session window.



Display I.5: Initial view of the dialog box for Column Statistics.



Display I.6: View of the dialog box for Column Statistics after selecting Mean and bringing up the variable list.



Statistic: ● Mean

Input variable: C2

Display I.7: Final view of the dialog box for Column Statistics.

Quite often, it is faster and more convenient to simply type your commands directly into the Session window. Sometimes, it is necessary to use the Session window approach, but for many commands the menu bar is available. So we now describe the use of commands in the Session window.

The basic structure of such a command with n arguments is

command name $E_1,E_2,...,E_n$

where E_i is the *ith* argument. Alternatively, we can write

command name E_1 E_2 ... E_n

if we don't want to type commas. Conveniently, if the arguments $E_1,E_2,...,E_n$ are consecutive columns in the worksheet, we have the following short-form

command name E_1-E_n

which saves even more typing and accordingly decreases our chance of making a typing mistake. If you are going to type a long list of arguments and you don't want them all on the same line, then you can type the *continuation symbol* & where you want to break the line and then hit Enter. Minitab responds with the prompt

CONT>

and you continue to type argument names. The command is executed when you hit Enter after an argument name without a continuation character following it.

Many commands can, in addition, be supplied with various subcommands that alter the behavior of the command. The structure for commands with subcommands is

command name $E_1 \ldots E_{n_1}$;
subcommand name $E_{n_1+1} \ldots E_{n_2}$;
$\vdots$
subcommand name $E_{n_{k-1}+1} \ldots E_{n_k}$.

Notice that when there are subcommands each line ends with a semicolon until the last subcommand, which ends with a period. Also, subcommands may have arguments. When Minitab encounters a line ending in a semicolon it expects a subcommand on the next line and changes the prompt to

```
SUBC >
```

until it encounters a period, whereupon it executes the command. If while typing in one of your subcommands you suddenly decide that you would rather not execute the subcommand — perhaps you realize something was wrong on a previous line — then type **abort** after the `SUBC >` prompt and hit Enter. As a further convenience, it is worth noting that you need to only type in the first four letters of any Minitab command or subcommand.

For example, to calculate the mean of column C2 in the worksheet marks we can use the **mean** command in the Session window, as in

```
MTB > mean c2
```

and we obtain the same output in the Session window as before.

There are two additional ways in which you can input commands to Minitab. Instead of typing the commands directly into the Session window, you can also type these directly into the Command Line Editor, which is available via Edit I Command Line Editor. Multiple commands can then be typed directly into a box that pops up and executed when the Submit Commands button is clicked. Output appears in the Session window. Also, many commands are available on a *toolbar* that lies just below the menu bar at the top of the Minitab window. There is a different toolbar depending upon which window is active. We give a brief discussion of some of the features available in the toolbar in later sections.

7 Entering Data into a Worksheet

There are various methods for entering data into a worksheet. The simplest approach is to use the *Data window* to enter data directly into the worksheet by clicking your mouse in a cell and then typing the corresponding data entry and hitting Enter. Remember that you can make a Data window active by clicking anywhere in the window or by using Windows in the menu bar. If you type any character that is not a number, Minitab automatically identifies the column containing that cell as a text variable and indicates that by appending T to the column name, e.g., C5-T in Display I.4. You do not need to append the T when referring to the column. Also, there is a *data direction arrow* in the upper left corner of the data window that indicates the direction the cursor moves after you hit Enter. Clicking on it alternates between row-wise and column-wise data entry. Certainly, this is an easy way to enter data when it is suitable. Remember, columns are variables and rows are observations! Also, you can have multiple data windows open and move data between them. Use the command File I New to open a new worksheet.

7.1 Importing Data

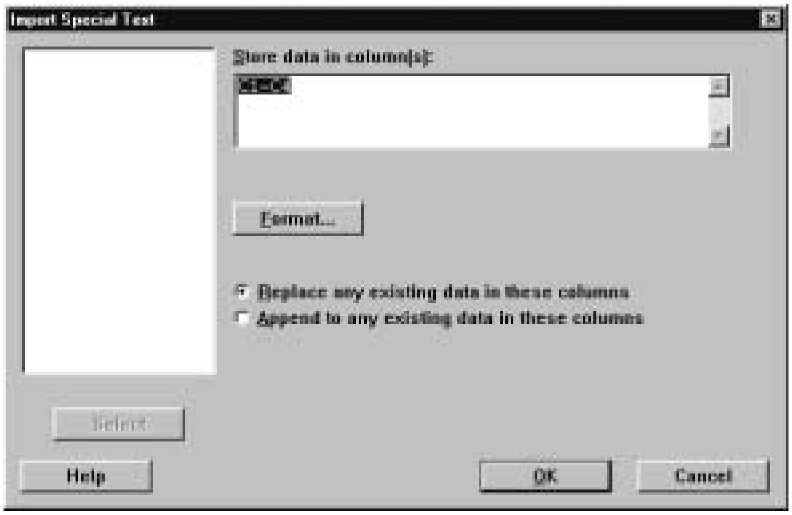
If your data is in an external file (not an `.mtw` file), you will need to use File I Other Files I Import Special Text to get the data into your worksheet. For example, suppose in the file `marks.txt` we have the following data recorded, just as it appears.

```
12389 81 85 78
97658 75 72 62
53546 77 83 81
55542 63 42 55
11223 71 82 67
77788 87 56 *
44567 23 45 35
32156 67 72 81
33456 81 77 88
67945 74 91 92
```

Each row corresponds to an observation, with the student number being the first entry, followed by the marks in the student's Statistics, Calculus, and Physics courses. These entries are separated by blanks.

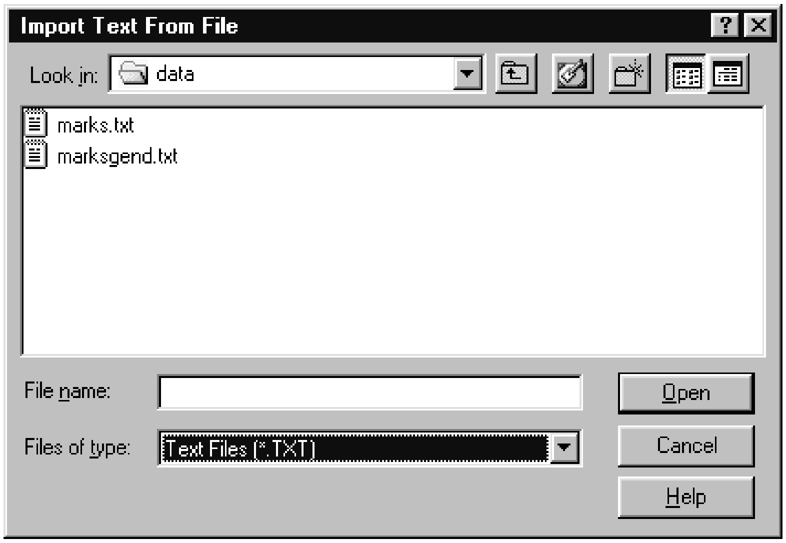
Notice the * in the sixth row of this data file. In Minitab, a * signifies a *missing numeric value,* i.e., a data value that for some reason is not available. Alternatively, we could have just left this entry blank. A *missing text value* is simply denoted by a blank. Special attention should be paid to missing values. In general, Minitab statistical analyses ignore any cases that contain missing data except that the output of the command will tell you how many cases were ignored because of missing data. It is important to pay attention to this information. If your data is riddled with a large number of missing values, your analysis may be based on very few observations — even if you have a large data set!

When data in such a file is *blank-delimited* like this it is very easy to read in. After the command File I Other Files I Import Special Text, we see the dialog box shown in Display I.8 minus C1–C4 in the Store data in column(s): box. We typed C1-C4 into this window to indicate that we want the data read in to be stored in these columns. Note that it doesn't matter if we use lower or upper case for the column names, as Minitab is not case sensitive. After clicking OK, we see the dialog box depicted in Display I.9, which we use to indicate from which file we want to read the data. Note that if your data is in `.txt` files rather than `.dat` files, you will have to indicate that you want to see these in the Files of type box by selecting Text Files or perhaps All Files. Clicking on `marks.txt` results in the data being read into the worksheet.



● Replace any existing data in these columns
○ Append to any existing data in these columns

Display I.8: Dialog box for importing data from external file.



Look in: data

marks.txt
marksgend.txt

File name:

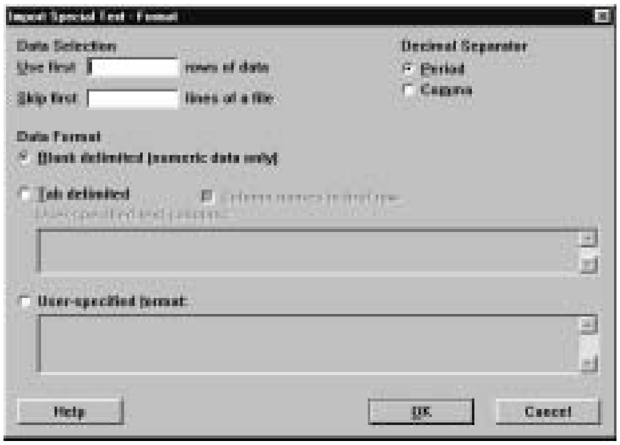
Files of type: Text Files (*.TXT)

Display I.9: Dialog box for selecting file from which data is to be read in.

Of course, this data set does not contain the text variable denoting the student's gender. Suppose that the file `marksgend.txt` contains the following data exactly as typed.

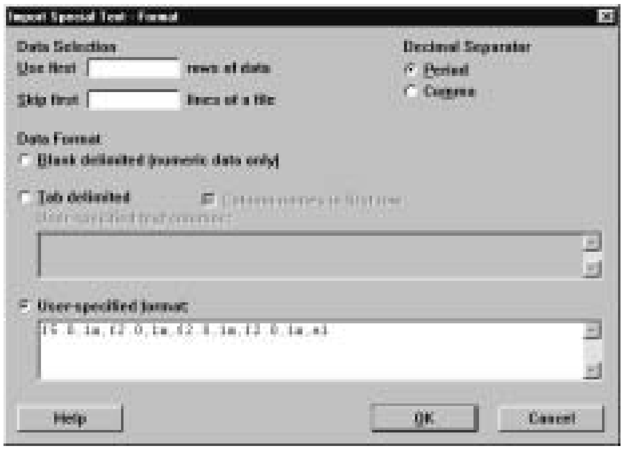
```
12389 81 85 78 m
97658 75 72 62 m
53546 77 83 81 f
55542 63 42 55 m
11223 71 82 67 f
77788 87 56  * f
44567 23 45 35 m
32156 67 72 81 m
33456 81 77 88 f
67945 74 91 92 f
```

As this file contains text data in the fifth column, we must tell Minitab how the data is *formatted* in the file. To access this feature we click on the Format button in the dialog box shown in Display I.8. This brings up the dialog box shown in Display I.10.



Display I.10: Initial dialog box for formatted input.

To indicate that we will specify the format, we click the radio button User-specified format and fill the particular format into the box as shown in Display I.11. The format statement says that we are going to read in the data according to the following rule: a numeric variable occupying 5 spaces and with no decimals, followed by a space, a numeric variable occupying 2 spaces with no decimals, a space, a numeric variable occupying 2 spaces with no decimals, a space, a numeric variable occupying 2 spaces with no decimals, a space, and a text variable occupying 1 space. This rule must be rigorously adhered to or errors will occur. So the rules you need to remember if you use formatted input are that `ak` indicates a text variable occupying `k` spaces, `kx` indicates `k` spaces, and `fk.l` indicates a numeric variable occupying `k` spaces, of which l are to the right of the decimal point. Note if a data value does not fill up the full number of spaces allotted to it in the format statement, it must be right justified in its field. Also, if a decimal point is included in the number, this occupies one of the spaces allocated to the variable and similarly for a negative or plus sign. There are many other features to formatted input that we will not discuss here. Use the Help button in the dialog box for information on these features. Finally, clicking on the OK button reads this data into a worksheet as depicted in Display I.4. Typically, we try to avoid the use of formatted input because it is somewhat cumbersome, but sometimes we must use it.


Import Special Text - Format

Data Selection
Use first □ rows of data
Skip first □ lines of a file

Decimal Separator
● Period
○ Comma

Data Format
○ Blank delimited (numeric data only)
○ Tab delimited
● User-specified format:
f5.0,1x,f2.0,1x,f2.0,1x,f2.0,1x,a1

Display I.11: Dialog box for formatted input with the format filled in.

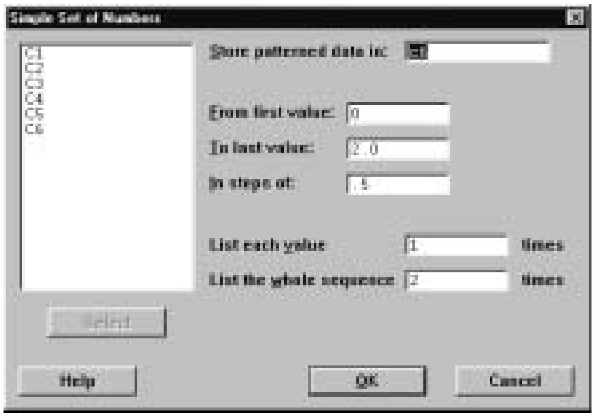
In the session environment, the **read** command is available for inputting data into a worksheet with capabilities similar to what we have described. For example, the commands

```
MTB >read c1-c4
DATA>12389 81 85 78
DATA>97658 75 72 62
DATA>53546 77 83 81
DATA>55542 63 42 55
DATA>11223 71 82 67
DATA>77788 87 56 *
DATA>44567 23 45 35
DATA>32156 67 72 81
DATA>33456 81 77 88
DATA>67945 74 91 92
DATA>end
 10 rows read.
```

place the first four columns into the `marks` worksheet. After typing `read c1-c4` after the `MTB >` prompt and hitting Enter, Minitab responds with the `DATA>` prompt, and we type each row of the worksheet in as shown. To indicate that there is no more data, we type `end` and hit Enter. Similarly, we can enter text data in this way but can't combine the two unless we use a **format** subcommand. We refer the reader to **help** for more description of how this command works.

7.2 Patterned Data

Often, we want to input *patterned data* into a worksheet. By this we mean that the values of a variable follow some determined rule. We use the command Calc I Make Patterned Data for this. For example, implementing this command with the entries in the dialog box depicted in Display I.12 adds a column C6 to the marks worksheet where the sequence $0, 0.5, 1.0, 1.5, 2.0$ is repeated twice. For this we entered 0 in the From first value box, a 2 in the To last value box, a .5 in the In steps of box, a 1 in the List each value box, and a 2 in the List the whole sequence box. Basically, we can start a sequence at any number m and successively increment this with any number $d > 0$ until the next addition would exceed the last value n prescribed, repeat each element l times, and finally repeat the whole sequence k times.



Store patterned data in: C7

From first value: 0

To last value: 2.0

In steps of: .5

List each value 1 times

List the whole sequence 2 times

Display I.12: Dialog box for making patterned data with some entries filled in.

There is some shorthand associated with patterned data that can be very convenient. For example, typing $m : n$ in a Minitab command is equivalent to typing the values $m, m + 1, \ldots, n$ when $m < n$ and $m, m - 1, ..., n$ when $m > n$ and m when $m = n$. The expression $m : n/d$, where $d > 0$, expands to a list as above but with the increment of d or $-d$, whichever is relevant, replacing 1 or -1. If $m < n$ then d is added to m until the next addition would exceed n and if $m > n$ then d is subtracted from m until the next subtraction would be lower than n. The expression $k(m : n/d)$ repeats $m : n/d$ for k times while $(m : n/d)l$ repeats each element in $m : n/d$ for l times. The expression $k(m : n/d)l$ repeats $(m : n/d)l$ for k times.

The **set** command is available in the session window to input patterned data. For example, suppose we want C6 to contain the 10 entries 1, 2, 3, 4, 5, 5, 4, 3, 2, 1. The command

```
MTB >set c6
DATA>1:5
DATA>5:1
DATA>end
```

does this. Also, we can add elements in parentheses. For example, the command

```
MTB >set c6
DATA>(1:2/.5 4:3/.2)
DATA>end
```

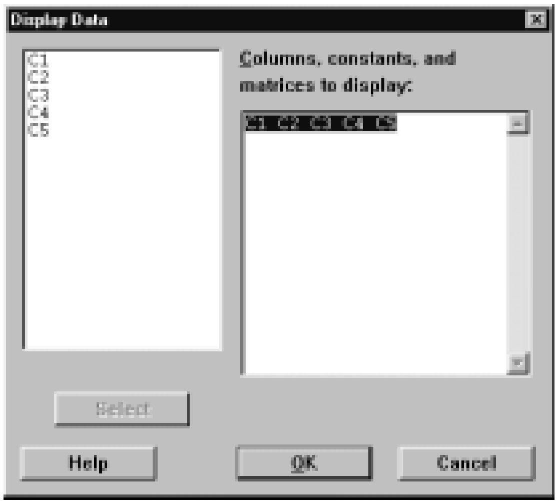
creates the column with entries 1.0, 1.5, 2.0, 4.0, 3.8, 3.6, 3.4, 3.2, 3.0. The multiplicative factors k and l can also be used in such a context. Obviously, there is a great deal of scope for entering patterned data with **set**. The general syntax of the set command is

set E_1

where E_1 is a column.

7.3 Printing Data in the Session Window

Once we have entered the data into the worksheet, we should always check that we have made the entries correctly. Typically, this means printing out the worksheet and checking the entries. The command Manip I Display Data will print the data you ask for in the Session window. For example, with the worksheet `marks` the dialog box pictured in Display I.13 causes the contents of this worksheet to be printed when we click on OK. We selected which variables to print by first clicking in the Columns, constants, and matrices to display box and then double clicking on the variables in the variable list on the left.



Columns, constants, and matrices to display: C1 C2 C3 C4 C5

Display I.13: Dialog box for printing worksheet in the Session window.

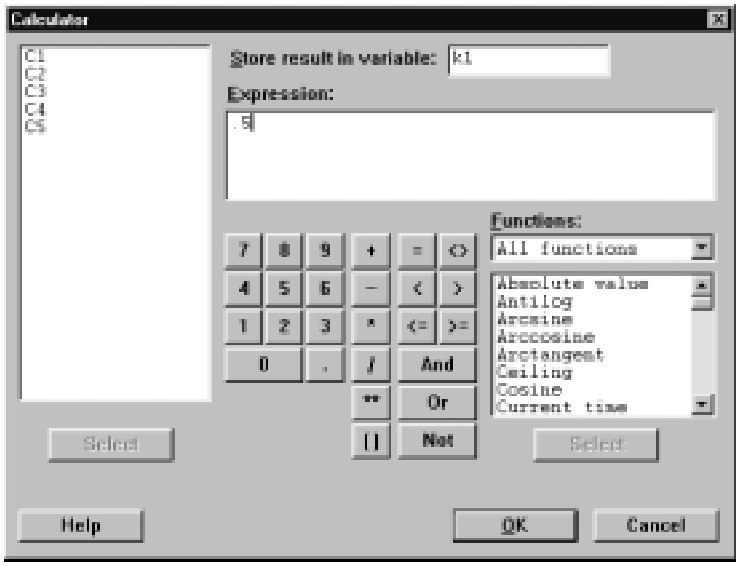
The **print** command is available in the Session window and is often convenient to use. The general syntax for the **print** command is

print E_1 ... E_m

where E_1, ..., E_m are columns and constants.

7.4 Assigning Constants

To enter constants, we use the Calc `I` Calculator command and fill in the dialog box appropriately. For example, suppose we want to assign the values `k1=.5`, `k2=.25` and `k3=.25` to the constants k1, k2, and k3. These could serve as weights to calculate a weighted average of the marks in the `marks` worksheet. Then the Calc `I` Calculator command leads to the dialog box displayed in Display I.14, where we have typed k1 into the Store result in variable box and the value .5 into the Expression box. Clicking on OK then makes the assignment. Note that we can assign text values to constants by enclosing the text in double quotes. We will talk about further features of Calculator later in this manual. Similarly, we assign values to k2 and k3.



Store result in variable: k1

Expression: .5

Functions: All functions

Display I.14: Filled in dialog box for assigning the constant k1 the value .5.

The **let** command is available in the Session window and is quite convenient. The following commands make this assignment and then we check, using the **print** command, that we have entered the constants correctly.

```
MTB >let k1=.5
MTB >let k2=.25
MTB >let k3=.25
MTB >print k1-k3
K1 0.500000
K2 0.250000
K3 0.250000
```

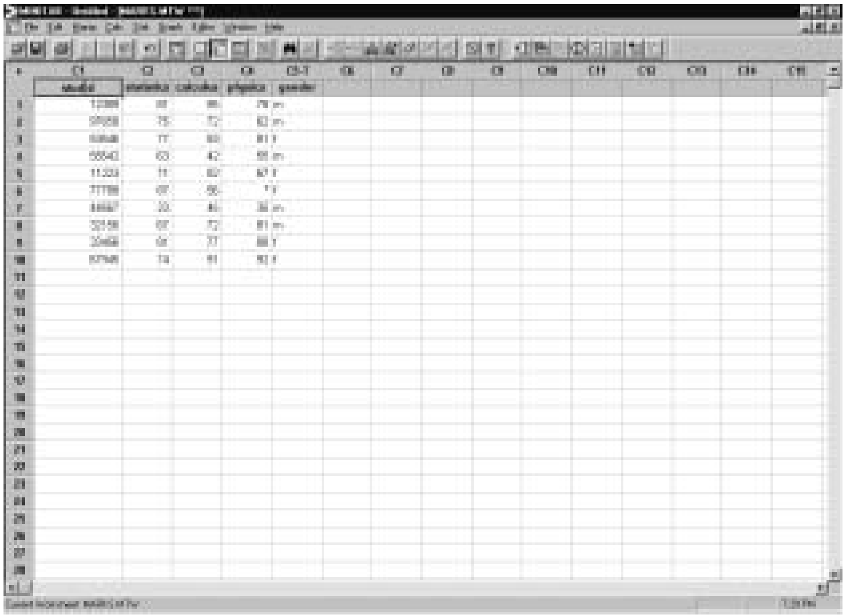
Also, we can assign constants text values. For example,

```
MTB >let k4=''result''
```

assigns K4 the value `result`. Note the use of double quotes.

7.5 Naming Variables and Constants

It often makes sense to give the columns and constants names rather than just referring to them as C1, C2, ..., K1, K2, etc. This is especially true when there are many variables and constants, as it would be easy to slip and use the wrong column in an analysis and then wind up making a mistake. To assign a name to a variable simply go to the blank cell at the top of the column in the worksheet corresponding to the variable and type in an appropriate name. For example, we have used `studid, statistics, calculus, physics,` and `gender` for the names of C1, C2, C3, C4, and C5, respectively, and these names appear in Display I.15.



↓	C1	C2	C3	C4	C5-T
	studid	statistics	calculus	physics	gender
1	12389	81	85	78	m
2	97658	75	72	82	m
3	53546	77	83	81	f
4	55542	63	42	55	m
5	11223	71	83	67	f
6	77788	67	55	*	f
7	44567	23	45	38	m
8	32156	87	72	81	m
9	20456	64	77	88	f
10	67945	74	91	92	f

Display I.15: Worksheet marks with named variables.

In the Session window, the **name** command is available for naming variables and constants. For example, the commands

```
MTB >name c1 'studid' c2 'stats' c3 'calculus' &
CONT>c4 'physics' c5 'gender' &
CONT>k1 'weight1' k2 'weight2' k3 'weight3'
```

give the names `studid` to C1, `stats` to C2, `calculus` to C3, `physics` to C4, `gender` to C5, `weight1` to K1, `weight2` to K2, and `weight3` to K3. Notice that we have made use of the continuation character & for convenience in typing in the full input to **name**. When using the variables as arguments just enclose the names in single quotes. For example,

```
MTB >print 'studid' 'calculus'
```

prints out the contents of these variables in the Session window.

Variable and constant names can be at most 31 characters in length, cannot include the characters # and ' and cannot start with a leading blank or *. Recall that Minitab is not case sensitive, so it does not matter if we use lower or upper case letters when specifying the names.

7.6 Information about a Worksheet

We can get information on the data we have entered into the worksheet by using the **info** command in the Session window. For example, we get the following results based on what we have entered into the `marks` worksheet so far.

```
MTB >info
  Column    Name        Count Missing
A C1        studid         10       0
  C2        stats          10       0
  C3        calculus       10       0
  C4        physics        10       1
A C5        gender         10       0
  Constant  Name       Value
  K1        weight1    0.500000
  K2        weight2    0.250000
  K3        weight3    0.250000
```

Notice that the **info** command tells us how many missing values there are and in what columns they occur and also the values of the constants.

This information can also be accessed directly from the *Project Manager window* via Window I Project Manager.

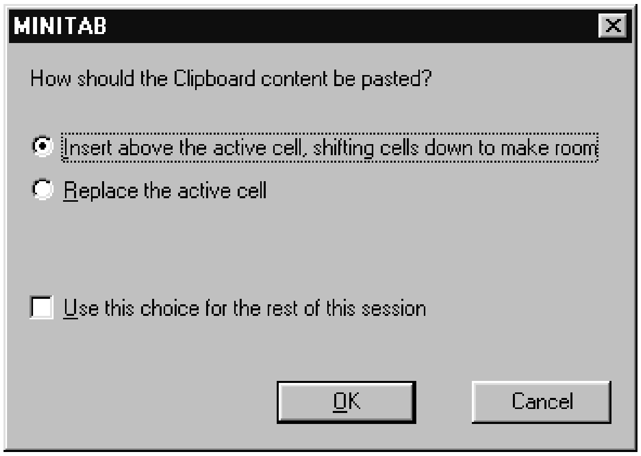
7.7 Editing a Worksheet

It often happens that after data entry we notice that we have made some mistakes or we obtain some additional information, such as more observations. So far, the only way we could change any entries in the worksheet or add some rows is to reenter the whole worksheet!

Editing the worksheet is straightforward because we simply change any cells by retyping their entries and hitting the Enter key. We can add rows and columns at the end of the worksheet by simply typing new data entries in the relevant cells. To insert a row before a particular row, simply click on any entry in that row and then the menu command Editor I Insert Rows. Fill in the blank entries in the new row. To insert a column before a particular column, simply click on any entry in that column and then the menu command Editor I Insert Columns. Fill in the blank entries in the new column. To insert a cell before a particular cell, simply click on any entry in that cell and the menu command Editor I Insert Cells. Fill in the blank entry in the new cell that appears in place of the original with all other cells in that column — and only that column — pushed down.

If you wish to clear a number of cells in a block, click in the cell at the start of the block, and holding the mouse key down, drag the cursor through the block so that it is highlighted in black. Click on the Cut Cells icon on the Minitab *taskbar,* and all the entries will be deleted. Cells immediately below the block move up to fill in the vacated places. A convenient method for clearing all the data entries in a worksheet, with the relevant Data window active, is to use the command Edit I Select All Cells, which causes all the cells to be highlighted, and click on the Cut Cells icon. Always save the contents of the current worksheet before doing this unless you are absolutely sure you don't need the data again. We discuss how to save the contents of a worksheet in I.8.1.

To copy a block of cells, click in the cell at the start of the block and, holding the mouse key down, drag the cursor through the block so that it is highlighted in black, but, instead of hitting the backspace key, use the command Edit I Copy Cells or click on the Copy Cells icon on the Minitab taskbar. The block of cells is now copied to your clipboard. If you not only want to copy a block of cells to your clipboard but remove them from the worksheet, use the command Edit I Cut Cells or the Cut Cells icon on the Minitab taskbar instead. Note that any cells below the removed block will move up to replace these entries. To paste the block of cells into the worksheet, click on the cell before which you want the block to appear or that is at the start of the block of cells you wish to replace and issue the command Edit I Paste Cells, or use the Paste Cells icon on the Minitab taskbar. A dialog box appears as in Display I.16, where you are prompted as to what you want to do with the copied block of cells. If you feel that a cutting or pasting was in error, you can undo this operation by using Edit I Undo Cut or Edit I Undo Paste, respectively, or use the Undo icon on the Minitab taskbar.



How should the Clipboard content be pasted?

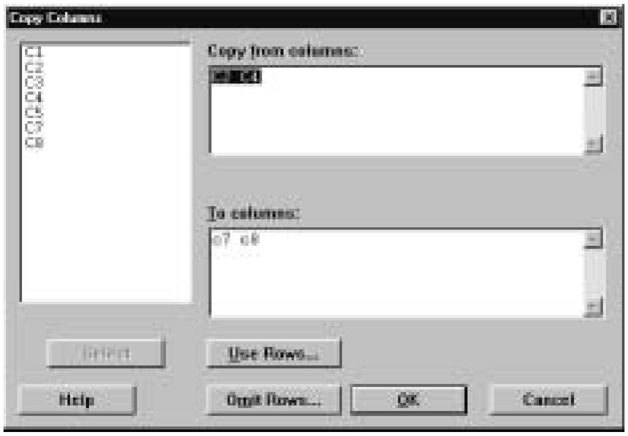
● Insert above the active cell, shifting cells down to make room

○ Replace the active cell

□ Use this choice for the rest of this session

Display I.16: Dialog box that determines how a block of copied cells is used, whether being inserted into a worksheet or replacing a block of cell of the same size.

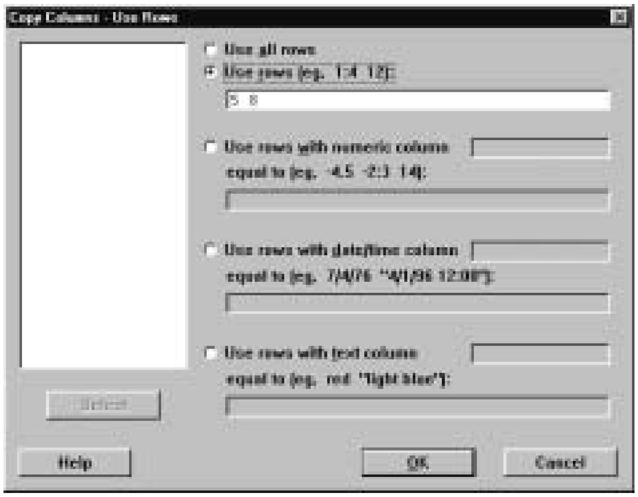
An alternative approach is available for copying operations using Manip I Copy Columns and filling in the dialog box appropriately. For example, suppose we want to copy all the entries in the `marks` worksheet in rows 5 and 8 of columns C2 and C4 and place these in columns C7 and C8. The dialog box shown in Display I.17 would result in all the entries in columns C2 and C4 being copied to C7 and C8. To prevent this, we click on the Use Rows button, which brings up the dialog box shown in Display I.18. Clicking on the Use rows radio button and filling in the associated box with the entries 5 and 8 specifies that only entries in the fifth and eighth rows will be copied. Clicking on the OK buttons in these dialog boxes then completes the operation.



Copy from columns: [illegible]

To columns: c7 c8

Display I.17: Dialog box for copying entries in columns and pasting them.


Copy Columns - Use Rows

○ Use all rows
● Use rows (eg. 1:4 12):
5 8

○ Use rows with numeric column
equal to (eg. -4.5 -2:3 14):

○ Use rows with date/time column
equal to (eg. 7/4/76 "4/1/96 12:00"):

○ Use rows with text column
equal to (eg. red "light blue"):

Display I.18: Dialog box to select rows from columns to be copied.

One can also delete selected rows from specified columns using Manip I Delete Rows and filling in the dialog box appropriately. Notice, however, that whenever we delete a cell, the contents of the cells beneath the deleted one in that column simply move up to fill the cell. The cell entry does not become missing; rather, cells at the bottom of the column become undefined! If you delete an entire row, this is not a problem because the rows below just shift up. For example, if we delete the third row then in the new worksheet, after the deletion, the third row is now occupied by what was formerly the fourth row. Therefore, you should be very careful, when you are not deleting whole rows, to ensure that you get the result you intended.

Note that if you should delete all the entries from a column, this variable is still in the worksheet, but it is empty now. If you wish to delete a variable and all its entries, this can be accomplished from Manip I Erase Variables and filling in the dialog box appropriately. This is a good idea if you have a lot of variables and no longer need some of them.

There are various commands in the Session window available for carrying out these editing operations. For example, the **restart** command in the Session window can be used to remove all entries from a worksheet. The **let** command allows you to replace individual entries. For example,

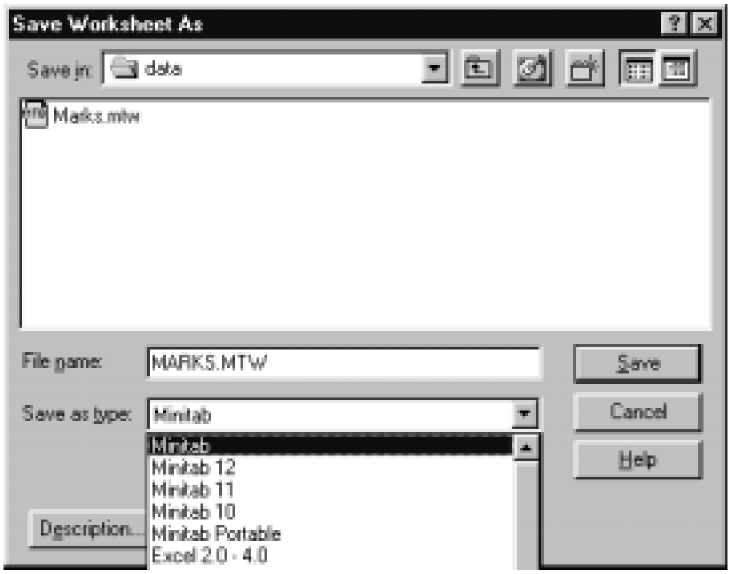
```
MTB > let c2(2)=3
```

assigns the value 3 to the second entry in the column C2. The **copy** command can be used to copy a block of cell from one place to another. The **insert** command allows you to insert rows or observations anywhere in the worksheet. The **delete** command allows you to delete rows. The **erase** command is available for the deletion of columns or variables from the worksheet. As it is more convenient to edit a worksheet by directly working on the worksheet and using the menu commands, we do not discuss these commands further here.

8 Saving, Retrieving, and Printing

Quite often, you will want to save the results of all your work in creating a worksheet. If you exit Minitab before you save your work, you will have to reenter everything. So we recommend that you always save. To use the commands of this section make sure that the Worksheet window of the worksheet in question is active.

Use File I Save Current Worksheet to save the worksheet with its current name, or the default name if it doesn't have one. If you want to provide a name or store the worksheet in a new location, then use File I Save Current Worksheet As and fill in the dialog box depicted in Display I.19 appropriately. The Save in box at the top contains the name of the folder in which the worksheet will be saved once you click on the Save button. Here the folder is called `data`, and you can navigate to a new folder using the Up One Level button immediately to the right of this box. The next button takes you to the Desktop and the third button allows you to create a subfolder within the current folder. The box immediately below contains a list of all files of type `.mtw` in the current folder. You can select the type of file to display by clicking on the arrow in the Save as type box, which we have done here, and click on the type of file you want to display that appears in the drop-down list. There are several possibilities including saving the worksheet in other formats, such as Excel. Currently, there is only one `.mtw` file in the folder `data` and it is called `marks.mtw`. If you want to save the worksheet with a different name, type this name in the File name box and click on the Save button.



Save Worksheet As

Save in: data

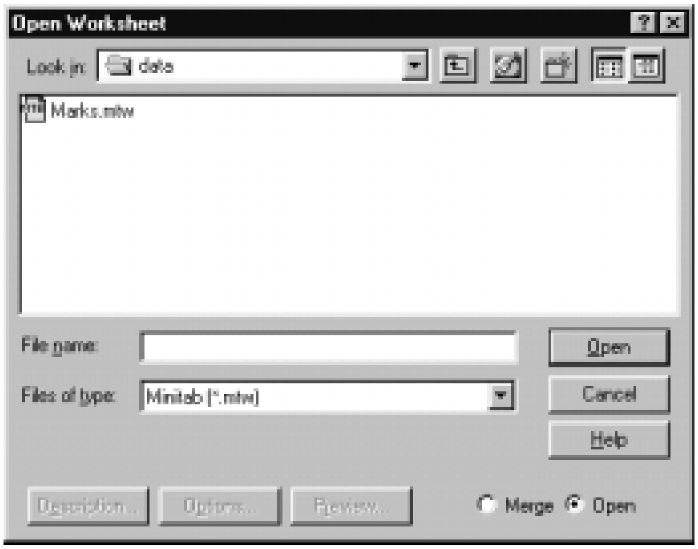
Marks.mtw

File name: MARKS.MTW

Save as type: Minitab

Display I.19: Dialog box for saving a worksheet.

To retrieve a worksheet, use File I Open Worksheet and fill in the dialog box as depicted in Display I.20 appropriately. The various windows and buttons in this dialog box work as described for the File I Save Current Worksheet As command, with the exception that we now type the name of the file we want to open in the File name box and click on the Open button.



Open Worksheet

Look in: data

Marks.mtw

File name:

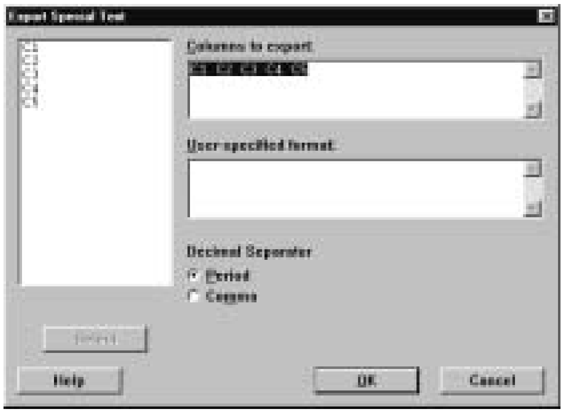
Files of type: Minitab (*.mtw)

○ Merge ● Open

Display I.20: Dialog box for retrieving a worksheet.

To print a worksheet, use the command File I Print Worksheet. The dialog box that subsequently pops up allows you to control the output in a number of ways.

It may be that you would prefer to write out the contents of a worksheet to an external file that can be edited by an editor or perhaps used by some other program. This will not be the case if we save the worksheet as an `.mtw` file as only Minitab can read these. To do this, use the command File I Other Files I Export Special Text, filling in the dialog box and specifying the destination file when prompted. For example, if we want to save the contents of the `marks` worksheet, this command results in the dialog box of Display I.21 appearing. We have entered all five columns into the Columns to export box and have not specified a format so the columns will be stored in the file with single blanks separating the columns. Clicking the OK button results in the dialog box of Display I.22 appearing. Here, we have typed in the name `marks.dat` to hold the contents. Note that while we have chosen a `.dat` type file, we also could have chosen a `.txt` type file. Clicking on the Save button results in a file `marks.dat` being created in the folder `data` with contents as displayed in Display I.23.

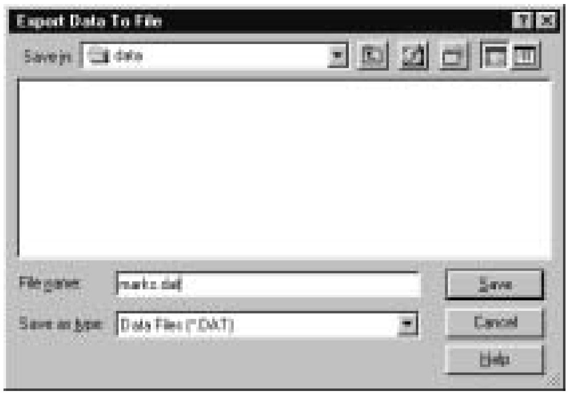


Columns to export: C1 C2 C3 C4 C5

User-specified format:

Decimal Separator
● Period
○ Comma

Display I.21: Dialog box for saving the contents of a worksheet to an external (non-Minitab) file.



Save in: data

File name: marks.dat

Save as type: Data Files (*.DAT)

Display I.22: Dialog box for selecting external file to hold contents of a worksheet.

```
12389    81    85    78   m
97658    75    72    62   m
53546    77    83    81   f
55542    63    42    55   m
11223    71    82    67   f
77788    87    56     *   f
44567    23    45    35   m
32156    67    72    81   m
33456    81    77    88   f
67945    74    91    92   f
```

Display I.23: Contents of the file marks.dat.

In the Session window, the commands **save** and **retrieve** are available for saving and retrieving a worksheet in the `.mtw` format and the command `write` is available for saving a worksheet in an external file. We refer the reader to **help** for a description of how these commands work.

9 Recording and Printing Sessions

Sometimes, it is useful — e.g., when you have to hand in an assignment — to maintain a record of all the commands you used, the output you obtained, and any comments you want to make on what you are doing in a Minitab session. Note that after executing a menu command the relevant Session window commands are automatically typed in the Session window.

To use the commands for saving or printing the Session window first make sure that the Session window is active. If you issue the menu command Editor I Output Editable first, you can edit the Session window contents before saving or printing its contents simply by typing or erasing text in the Session window. You can turn this feature off using the same command. To save the contents of a Session window use File I Save Session Window As and fill in the dialog box appropriately. Note that the saved file is in the `.txt` format unless you make a different choice in the Save as type box. To print the contents of the Session window use File I Print Session Window.

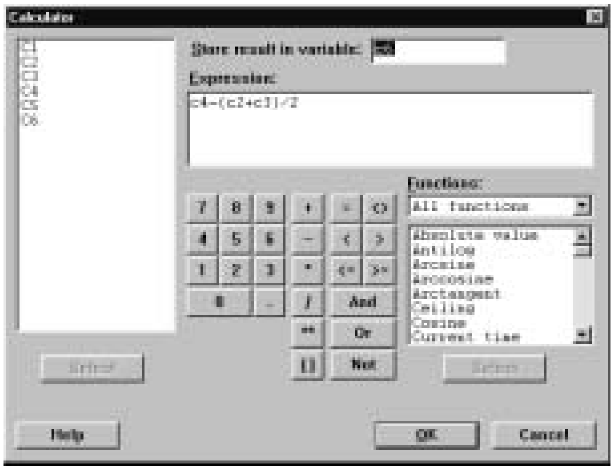
In the Session window, the **outfile** command is available for recording the full or partial contents of a Minitab session. We refer the reader to **help** for a description of how this command works.

10 Mathematical Operations

When carrying out a data analysis a statistician is often called upon to transform the data in some way. This may involve applying some simple transformation to a variable to create a new variable — e.g., take the natural logarithm of every grade in the `marks` worksheet — to combining several variables together to form a new variable — e.g., calculate the average grade for each student in the `marks` worksheet. In this section, we present some of the ways of doing this.

10.1 Arithmetical Operations

Simple arithmetic can be carried out on the columns of a worksheet using the arithmetical operations of addition +, subtraction −, multiplication *, division /, and exponentiation ** via the Calc I Calculator command. When columns are added together, subtracted one from the other, multiplied together, divided one by the other (make sure there are no zeros in the denominator column), or one column exponentiates another, these operations are always performed component-wise. For example, C1*C2 means that the *ith* entry of C1 is multiplied by the *ith* entry of C2; etc. Also, make sure that the columns on which you are going to perform these operations correspond to numeric variables! While these operations have the order of precedence **, */, +−, parentheses ( ) can and should be used to ensure an unambiguous result. For example, suppose in the `marks` worksheet we want to create a new variable by taking the average of the Statistics and Calculus grades and then subtracting this from the Physics grade and placing the result in C6. Filling in the dialog box, corresponding to Calc I Calculator, as shown in Display I.24 accomplishes this when we click on the OK button.



Store result in variable: c6

Expression: c4-(c2+c3)/2

Display I.24: Dialog box for carrying out mathematical calculations.

Note that we can either type the relevant expression into the Expression box or use the buttons and double clicking on the relevant columns. Further, we type the column where we wish to store the results of our calculation in the Store result in variable box. These operations are done on the corresponding entries in each column; corresponding entries in the columns are operated on according to the formula we have specified, and a new column of the same length containing all the outcomes is created. Note that the sixth entry in C6 will be * — missing — because this entry was missing for C4.

These kinds of operations can also be carried out directly in the Session window using the **let** command, and in some ways this is a simpler approach. For example, the session command

```
MTB >let c6=c4-(c2+c3)/2
```

accomplishes this.

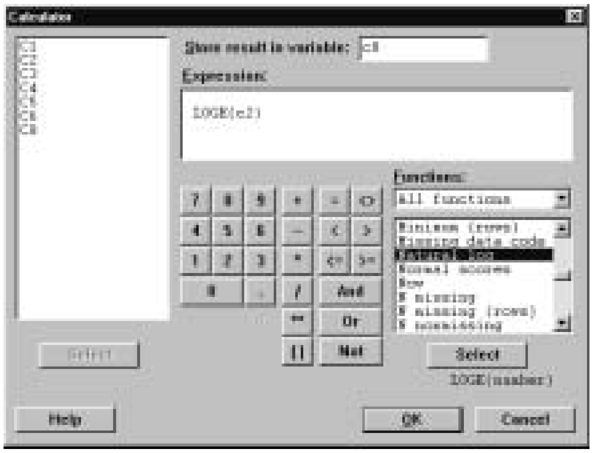
We can also use these arithmetical operations on the constants K1, K2, etc., and numbers to create new constants or use the constants as *scalars* in operations with columns. For example, suppose that we want to compute the weighted average of the Statistics, Calculus, and Physics grades where Statistics gets twice the weight of the other grades. Recall that we created, as part of the marks worksheet, the constants weight1 = .5, weight2 = .25, and weight3 = .25 in K1, K2, and K3, respectively. So this weighted average is computed via the command

```
MTB >let c7='weight1'*'stats'+'weight2'*'calculus'&
CONT>+'weight3'*'physics'
```

and the result is placed in C7. We have used the continuation character & for convenience in this computation. Alternatively, we could have used the Calc I Calculator command as above for this.

10.2 Mathematical Functions

Various mathematical functions are available in Minitab. For example, suppose we want to compute the natural logarithm of the Statistics mark for each student. Using the Calc I Calculator command with the dialog box as in Display I.25 accomplishes this.



Store result in variable: c8

Expression: LOGE(c2)

Functions: All functions

LOGE(number)

Display I.25: Dialog box for mathematical calculations illustrating the use of the natural logarithm function.

A complete list of such functions is given in the Functions window when All functions is in the window directly above the list.

The same result can be obtained using the session command **let** and the natural logarithm function **loge**. For example,

```
MTB >let c8=loge(c2)
```

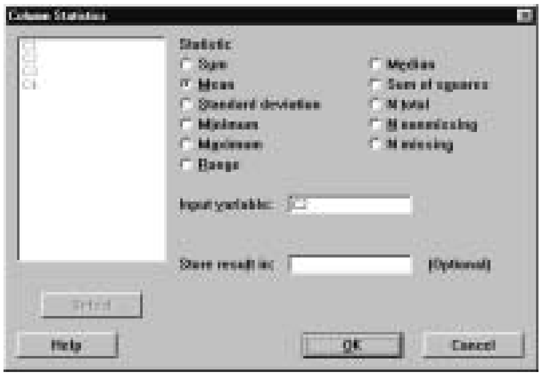
calculates the natural log of every entry in c2 and places the results in C8. There are a number of such functions and a complete list is provided in Appendix B.1. These functions can be applied to numbers as well as constants. If you want to know the sine of the number 3.4, then

```
MTB >let k4=sin(3.4)
MTB >print k4
K4 -0.255541
```

gives the value.

10.3 Column and Row Statistics

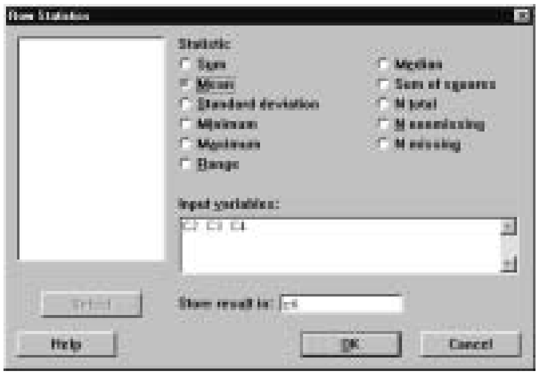
There are various *column statistics* that compute a single number from a column by operating on all of the elements in a column. For example, suppose that we want the mean of all the Statistics marks, i.e., the mean of all the entries in C2. The command Calc I Column Statistics produces the dialog box of Display I.26 where we have selected Mean as the particular statistic to compute and C2 as the column to use. Clicking OK causes the mean of column C2 to be printed in the Session window.



Display I.26: Dialog box for computing column statistics.

If we want to, we can store this result in a constant or column by making an appropriate entry in the Store result in box. We see from the dialog box that there are a number of possible statistics that can be computed.

We can also compute statistics row-wise. One difference with column statistics is that these must be stored. For example, suppose we want to compute the average of the Statistics, Calculus, and Physics marks. The command Calc I Row Statistics produces the dialog box shown in Display I.27 where we have placed C2, C3, and C4 into the Input variables box and c6 into the Store result in box.



Display I.27: Dialog box for computing row statistics.

It is also possible to compute column statistics using session commands. For example,

```
MTB >mean(c2)
MEAN = 69.900
```

computes the mean of c2. If we want to save the value for subsequent use, then the command

```
MTB >let k1=mean(c2)
```

does this. The general syntax for column statistic commands is

column statistic name(E_1)

where the operation is carried out on the entries in column E_1, and output is written to the screen unless it is assigned to a constant using the **let** command. See Appendix B.2 for a list of all the column statistics available.

Also, for most column statistics there are versions that compute *row statistics,* and these are obtained by placing **r** in front of the column statistic name. For example,

```
MTB >rmean(c2 c3 c4 c6)
```

computes the mean of the corresponding entries in C2, C3, and C4 and places the result in C6. The general syntax for row statistic commands is

row statistic name($E_1 \ldots E_m$ E_{m+1})

where the operations are carried out on the rows in columns $E_1, \ldots, E_m$, and the output is placed in column E_{m+1}. See Appendix B.3 for a list of all the row statistics available.

10.4 Comparisons and Logical Operations

Minitab also contains the following comparison and logical operators.

Comparison Operators	**Logical Operators**
equal to =, **eq**	&, **and**
not equal to <>, **ne**	\, **or**
less than <, **lt**	~, **not**
greater than >, **gt**	
less than or equal to <=, **le**	
greater than or equal to >=, **ge**	

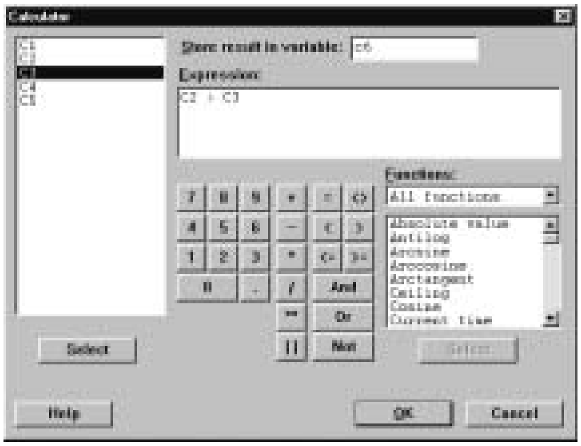
Notice that there are two choices for these operators; for example, use either the symbol >= or the mnemonic **ge.**

The comparison and logical operators are useful when we have simple questions about the worksheet that would be tedious to answer by inspection. This feature is particularly useful when we are dealing with large data sets. For example, suppose that we want to count the number of times the Statistics grade was greater than the corresponding Calculus grade in the `marks` worksheet. The command Calc I Calculator gives the dialog box shown in Display I.28 where we have put `c6` in the Store result in variable box and `c2 > c3` in the Expression box. Clicking on the OK button results in the *ith* entry in C6 containing a 1 if the *ith* entry in C2 is greater than the *ith* entry in C3, i.e., the comparison is true, and a 0 otherwise. In this case, C6 contains the entries: 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, which the worksheet in Display I.4 verifies as appropriate. If we use Calc I Calculator to calculate the sum of the entries in C6, we will have computed the number of times the Statistics grade is greater than the Calculus grade.

These operations can also be simply carried out using session commands. For example,

```
MTB >let c6=c2>c3
MTB >let k4=sum(c6)
MTB >print k4
K4 4.00000
```

accomplishes this.



Store result in variable: c6

Expression:
C2 > C3

Display I.28: Dialog box for comparisons.

The logical operators combine with the comparison operators to allow more complicated questions to be asked. For example, suppose we wanted to calculate the number of students whose Statistics mark was greater than their Calculus mark and less than or equal to their Physics mark. The commands

```
MTB >let c6=c2>c3 and c2<=c4
MTB >let k4=sum(c6)
MTB >print k4
K4 1.00000
```

accomplish this. In this case, both conditions `c2>c3` and `c2<=c4` have to be true for a 1 to be recorded in C6. Note that the observation with the missing Physics mark is excluded. Of course, we can also implement this using Calc I Calculator and filling in the dialog box appropriately.

Text variables can be used in comparisons where the ordering is alphabetical. For example,

```
MTB >let c6=c5<''m''
```

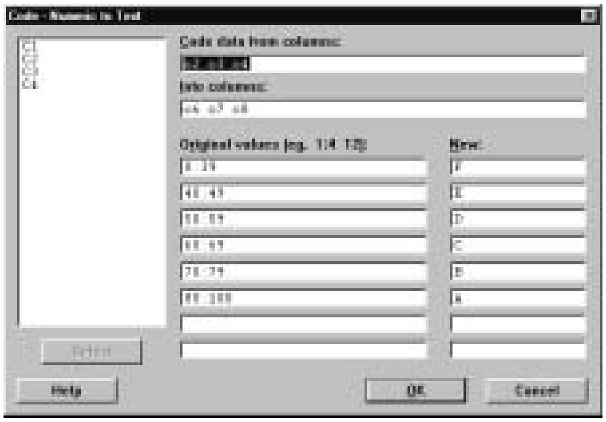
puts a 1 in C6 whenever the corresponding entry in C5 is alphabetically smaller than `m`.

11 Some More Minitab Commands

In this section we discuss some commands that can be very helpful in certain applications. We will make reference to these commands at appropriate places throughout the manual. It is probably best to wait to read these descriptions until such a context arises.

11.1 Coding

The Manip I Code command is used to recode columns. By this we mean that data entries in columns are replaced by new values according to a coding scheme that we must specify. You can recode numeric into numeric, numeric into text, text into numeric, or text into text by choosing an appropriate subcommand. For example, suppose in the `marks` worksheet we want to recode the grades in C2, C3, and C4 so that any mark in the range 0–39 becomes an F, every mark in the range 40–49 becomes an E, every mark in the range 50–59 becomes a D, every mark in the range 60–69 becomes a C, every mark in the range 70–79 becomes a B, every mark in the range 80–100 becomes an A, and the results are placed in columns C6, C7, and C8, respectively. Then the command Manip I Code I Numeric to Text brings up the dialog box shown in Display I.29. The ranges for the numeric values to be recoded to a common text value are typed in the Original values box, and the new values are typed in the New box. Note that we have used a shorthand for describing a range of data values as discussed in section 7.2. Because the sixth entry of C4 is *, i.e., it is missing, this value is simply recoded as a blank. You can also recode missing values by including * in one of the Original values boxes. If a value in a column is not covered by one of the values in the Original values boxes, then it is simply left the same in the new column.



Code data from columns: c2 c3 c4

Into columns: c6 c7 c8

Original values (eg, 1:4 12):	New:
0:39	F
40:49	E
50:59	D
60:69	C
70:79	B
80:100	A

Display I.29: Dialog box for recoding numeric values to text values.

Note that this menu command restricts the number of new code values to 8. The session command **code** allows up to 50 new codes. For example, suppose in the `marks` worksheet we want to recode the grades in C2, C3, and C4 so that any mark in the range 0–9 becomes a 0, every mark in the range 10–19 becomes 10, etc., and the results are placed in columns C6, C7, and C8. The following command

```
MTB >code(0:9) to 0 (10:19) to 10 (20:29) to 20 (30:39) to 30 &
CONT>(40:49) to 40 (50:59) to 50 (60:69) to 60 (70:79) to 70 &
CONT>(80:89) to 80 (90:99) to 90 for C2-C4 put in C6-C8
```

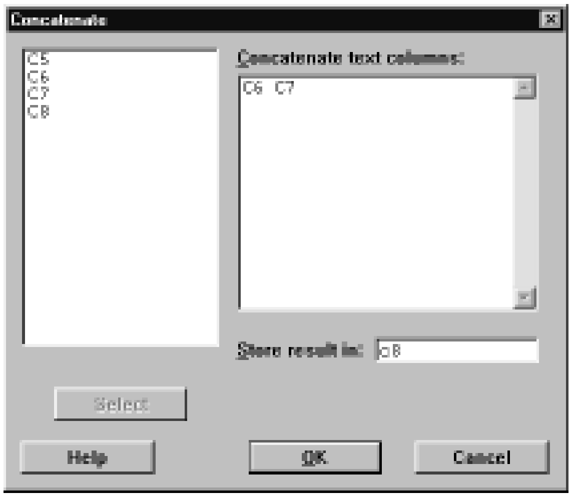
accomplishes this. Note the use of the continuation symbol &, as this is a long command. The general syntax for the **code** command is

code (V_1) to code$_1$ ... (V_n) to code$_n$ for E_1 ... E_m put in E_{m+1} ... E_{2m}

where V_i denotes a set of possible values and ranges for the values in columns E_1 ... E_m that are all coded as the number code$_i$, and the results of this coding are placed in the columns E_{m+1} ... E_{2m}, i.e., the recoded E_1 is placed in E_{m+1}, etc.

11.2 Concatenating Columns

The Manip I Concatenate combines two or more text columns into a single text column. For example, if C6 contains `m`, `m`, `m`, `f`, `f`, reading first to last entry, and C7 contains `to`, `ta`, `ti`, `to`, `ta`, then the entries in the Manip I Concatenate dialog box shown in Display I.30 result in a new text column C8 containing the entries `mto`, `mta`, `mti`, `fto`, `fta`.



Concatenate text columns: C6 C7

Store result in: C8

Display I.30: Dialog box for concatenating text columns.

In the session environment, the **concatenate** command is available for this operation. The general syntax of the **concatenate** command is

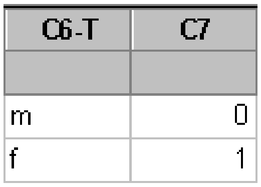
concatenate E_1 ... E_m in E_{m+1}

where E_1, ..., E_m, are text columns, and E_{m+1} is the target text column.

11.3 Converting Data Types

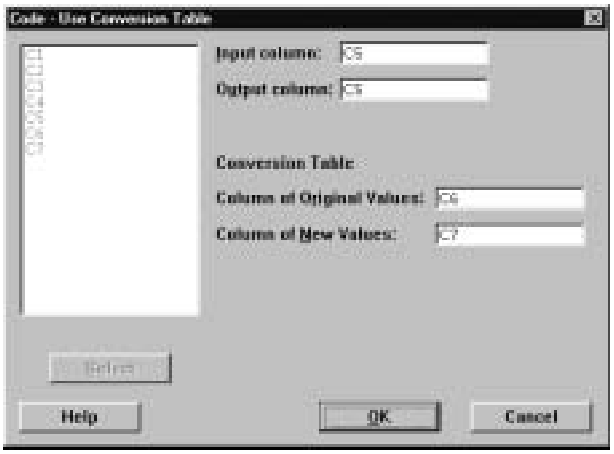
The Manip I Code I Use Conversion Table command is used to change text data into numeric data and vice versa. As dealing with text data is a bit more difficult in Minitab, we recommend either converting text data to numeric before input or using this command after input to do this.

For example, in the worksheet marks suppose we want to change the gender variable from text, with male and female denoted by m and f, respectively, to a numerical variable with male denoted by 0 and female by 1. To do this, we must first set up a *conversion table.* The conversion table comprises two columns in the worksheet, where one column is text and contains the text values used in the text column, and the second column is numeric and contains the numerical values that you want these changed into. For example, suppose we have entered columns C6 and C7 in the marks worksheet, as shown in Display I.31. The Manip I Code I Use Conversion Table command produces the dialog box shown in Display I.32, where we have indicated that we want to convert the text column C5 into a numeric column and that each m should become a 0 and each f should become a 1.



C6-T	C7
m	0
f	1

Display I.31: Columns c6 and c7 in the marks worksheet as a conversion table.



Code - Use Conversion Table

Input column: C5

Output column: C5

Conversion Table

Column of Original Values: C6

Column of New Values: C7

Display I.32: Dialog box for converting text column c5 of the marks worksheet into a numeric column with the conversion table given in columns c6 and c7.

The general syntax for the corresponding session command **convert** is

convert E_1 E_2 E_3 E_4

where E_1, E_2 are the columns containing the conversion table, E_3 is the column to be converted and E_4 is the column containing the converted column.

11.4 History

Minitab keeps a record of the commands you have used and the data you have input in a session. This information can be obtained in the History folder of the Project Manager window. The commands can be copied from wherever they are listed and pasted into the Session window to be reexecuted, so that a number of commands can be executed at once without retyping. These commands can be edited before being executed again. This is very helpful when you have implemented a long sequence of commands and realize that you made an error early on. Note that even if you use the menu commands, a record is kept only of the corresponding session commands.

The **journal** command is available in the Session window if you want to keep a record of the commands in an external file. For example,

```
MTB >journal 'comm1'
Collecting keyboard input(commands and data)in file:
                                                     comm1.MTJ
MTB >read c1 c2 c3
DATA>1 2 3
DATA>end
 1 rows read.
MTB >nojournal
```

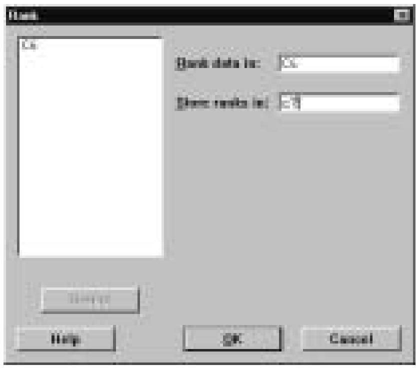
puts

```
read c1 c2 c3
1 2 3
end
nojournal
```

into the file `comm1.mtj`. The history is turned off as soon as the **nojournal** command is typed.

11.5 Computing Ranks

Sometimes, we want to compute the *ranks* of the numeric values in a column. The rank r_i of the *ith* value in a column is a value that reflects its relative size in the column. For example, if the *ith* value is the smallest value then r_i = 1, if it is the third smallest then r_i = 3, etc. If values are the same, i.e., *tied,* then each value receives the average rank. To calculate the ranks of the entries in a column we use the Manip I Rank command. For example, suppose that C6 contains the values 6, 4 , 3, 2, 3, 1. Then the Manip I Rank command brings up the dialog box in Display I.33, which is filled in so that the ranks of the entries in C6 are placed in C7. In this case, the ranks are 6.0, 5.0, 3.5, 2.0, 3.5, and 1.0, respectively.



Rank data in: C6

Store ranks in: C7

Display I.33: Dialog box for computing ranks.

The syntax of the corresponding session command **rank** is

rank E_1 E_2

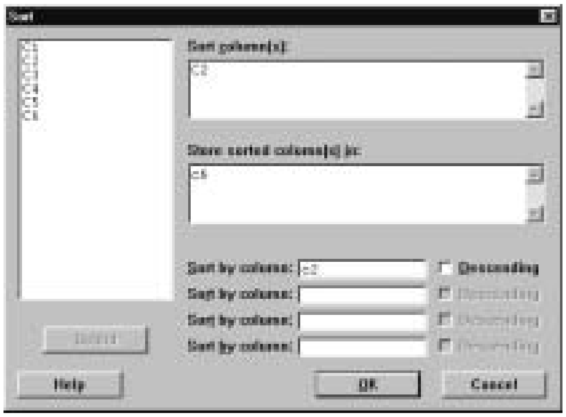
where E_1 is the column whose ranks we want to compute, and E_2 is the column that will hold the computed ranks.

11.6 Sorting Data

It often occurs as part of a data analysis that we want to *sort* a column so that its values ascend from smallest to largest or descend from largest to smallest. Note that ordering here could refer to numerical order or alphabetical order, so we also consider ordering text columns. Also, we may want to sort all the rows contained in some subset of the columns in the worksheet *by* a particular column. The Manip I Sort command allows us to carry out these tasks.

For example, suppose that we want to sort the entries in C2 in the `marks` worksheet — the Statistics grades — from smallest to largest and place the sorted values in C6. Then the Manip I Sort command brings up the dialog box shown in Display I.34, where the Sort column(s) box contains the column C2 to be sorted, the Store sorted column(s) In box contains C6, where we will store the sorted column, and C2 is also placed in the Sort by column box. This command results in C6 containing 23, 63, 67, 71, 74, 75, 77, 81, 81, 87. If we had clicked the Descending box, the order of appearance of these values in C6 would have been reversed.

If we had placed another column in the Sort by column box, say C5, then C5 would have been sorted with the values in C2 carried along and placed in C6, i.e., the values in C2 would be sorted *by* the values in C5. So all the Statistics marks of females, in the order they appear in C2 will appear in C6 first and then the Statistics marks of males. For example, replacing C2 by C5 in this box would result in the values in C6 becoming 77, 71, 87, 81, 74, 81, 75, 63, 23, 67. If we fill in the next Sort by column box with another column, say C3, then the values in C2 are sorted first by gender and then within gender by the values in C3.



Sort column(s): C2

Store sorted column(s) in: c6

Sort by column: c2 □ Descending

Sort by column: □ Descending

Sort by column: □ Descending

Sort by column: □ Descending

Display I.34: Dialog box for sorting.

The general syntax of the corresponding session command **sort** is

sort E_1 $E_2 \ldots E_m$ $E_{m+1} \ldots E_{2m}$

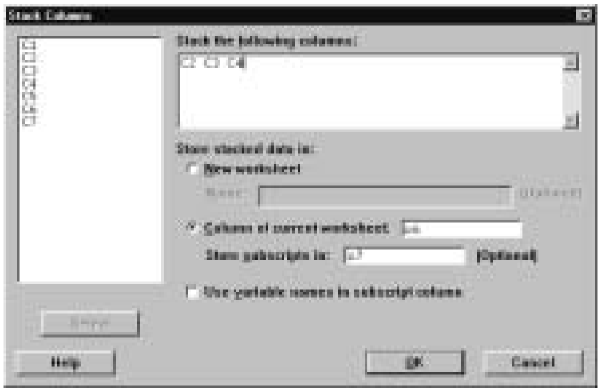
where E_1 is the column to be sorted, and E_2, ..., E_m are carried along with the results placed in columns E_{m+1}, ..., E_{2m}. Note that this sort can also be accomplished using the **by** subcommand, where the general syntax is

sort E_1 $E_2 \ldots E_m$ $E_{m+1} \ldots E_{2m}$;
by $E_{2m+1} \ldots E_n$.

where now we sort by columns E_{2m+1}, ..., E_n, sorting first by E_{2m+1}, then E_{2m+2}, etc., carrying along E_1, ..., E_m and placing the result in E_{m+1}, ..., E_{2m}. The **descending** subcommand can also be used to indicate which sorting variables we want to use in descending order rather than ascending order.

11.7 Stacking and Unstacking Columns

The Manip I Stack command is used to literally stack columns one on top of the other or unstack a column into separate columns. For example, in the marks worksheet the Manip I Stack I Stack Columns command brings up the dialog box shown in Display I.35, which has been filled in to stack columns C2, C3, and C4 into C6 with the values in C2 first, followed by the values in C3 and then the values in C4. In C7 we have stored an index which indicates that column each value in C6 came from with a 1 every time a value came from C2, a 2 every time a value came from C3, and a 3 every time a value came from C4. It is not necessary to create such an index.



Stack the following columns: C2 C3 C4

Store stacked data in:

○ New worksheet

Name: (Optional)

● Columns of current worksheet: C6

Store subscripts in: C7 (Optional)

□ Use variable names in subscript column

Display I.35: Dialog box for stacking columns.

In the Session window, this same result can be obtained using the **stack** command. The general syntax for the **stack** command is given by

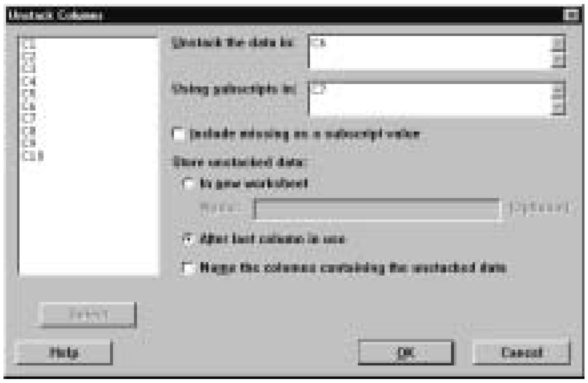
stack $E_1 E_2 \ldots E_m$ into E_{m+1}

where E_1, E_2, ..., E_m denote the columns or constants to be stacked one on top of the other, starting with E_1, and with the result placed in column E_{m+1}. If we want to keep an index of where the values came from, then use the subcommand

subscripts E_{m+2}

which results in index values being stored in column E_{m+2}.

To unstack values in a column by the values in an index column we use the Manip I Unstack command. For example, given the columns C6 and C7 of the `marks` worksheet as described above, the dialog box shown in Display I.36 unstacks C6 into three columns by the values in C7. The three columns are C8, C9, and C10. Note that they are identical to columns C2, C3, and C4, respectively. We must always specify a column containing the subscripts when unstacking a column.



Unstack Columns

Unstack the data in: C6

Using subscripts in: C7

□ Include missing as a subscript value

Store unstacked data:

○ In new worksheet

Name: (Optional)

● After last column in use

□ Name the columns containing the unstacked data

Display I.36: Dialog box for unstacking columns.

The general syntax for the corresponding session command **unstack** is

unstack E_1 into $E_2 \ldots E_m$;
subscripts E_{m+1}.

where E_1 is the column to be unstacked, E_2, ..., E_m are the columns and constants to contain the unstacked column, and E_{m+1} gives the subscripts 1, 2, ... that indicate how E_1 is to be unstacked.

Note that it is also possible to simultaneously unstack blocks of columns. We refer the reader to **help** or Help for information on this.

12 Exercises

1. The following data give the Hi and Low trading prices in Canadian dollars for various stocks on a given day on the Toronto Stock Exchange. Create a worksheet, giving the columns the same variable names, using any of the methods discussed in I.7. Be careful to ensure that the value of the variable `stock` starts with a letter. Print the worksheet to check that you have successfully entered it. Save the worksheet giving it the name `stocks`.

Stock	**Hi**	**Low**
ACR	7.95	7.80
MGI	4.75	4.00
BLD	112.25	109.75
CFP	9.65	9.25
MAL	8.25	8.10
CM	45.90	45.30
AZC	1.99	1.93
CMW	20.00	19.00
AMZ	2.70	2.30
GAC	52.00	50.25

2 Retrieve the worksheet `stocks` created in Exercise 1. Change the `Low` value in the stock MGI to 3.95. Calculate the average of the `Hi` and `Low` prices for all the stocks, and save this in a column called `average`. Calculate the average of all the `Hi` prices, and save this in a constant called `avhi`. Similarly, do this for all the `Low` prices, and save this in a constant called `avlo`. Save the worksheet using the same name. Write all the columns out to a file called `stocks.dat`. Print the file `stocks.dat` on your system printer.

3 Retrieve the worksheet created in Exercise 2. Using the Minitab commands discussed in I.10, calculate the number of stocks in the worksheet whose `average` is greater than $5.00 and less than or equal to $45.00.

4 Using the worksheet created in Exercise 2, insert the following stocks at the beginning of the worksheet.

Stock	**Hi**	**Low**
CLV	1.85	1.78
SIL	34.00	34.00
AC	14.45	14.05

Delete the variable `average`. Save the worksheet.

5 Using the worksheet created in Exercise 4, sort the stocks into alphabetical order. Calculate the ranks of the individual stocks based on their `Hi` price, and save the ranking in a new column. Save the worksheet.

6 Using the worksheet created in Exercise 5, calculate the average `Hi` price of all the stocks beginning in A.

7 Using the worksheet created in Exercise 5, recode all the `Low` prices in the range $0–9.99 as 1, in the range $10–39.99 as 2, and greater than or equal to $40 as 3, and save the recoded variable in a new column.

8 Using patterned data input, place the values from -10 to 10 in increments of .1 in C1. For each of the values in C1, calculate the value of the quadratic polynomial $2x^2 + 4x - 3$ (i.e., substitute the value in each entry in C1 into this expression) and place these values in C2. Using Minitab commands and the values in C1 and C2, estimate the point in the range from -10 to 10 where this polynomial takes its smallest value and what this smallest value is. Using Minitab commands and the values in C1 and C2 estimate the points in the range from -10 to 10, where this polynomial is closest to 0.

9 Using patterned data input, place values in the range from 0 to 5 using an increment of .01 in C1. Calculate the value of $1 - e^{-x}$ for each value in C1, and place the result in C2. Using Minitab commands, find the largest value in C1 where the corresponding entry in C2 is less than or equal to .5. Note that e^{-x} corresponds to the **exponentiate** command (see Appendix B.1) evaluated at $-x$.

10 Using patterned data input, place values in the range from -4 to 4 using an increment of .01 in C1. Calculate the value of

$$\frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$$

for each value in C1, and place the result in C2, where $\pi = 3.1415927$. Using **parsums** (see Appendix B.1), calculate the partial sums for C2, and place the result in C3. Multiply C3 times .01. Find the largest value in C1 such that the corresponding entry in C3 is less than or equal to .25.

Part II

Minitab for Data Analysis

Chapter 1

Looking at Data – Distributions

New Minitab commands discussed in this chapter

Calc I Probability Distributions I Normal
File I Open Graph
File I Save Graph As
Graph I Boxplot
Graph I Chart
Graph I Dotplot
Graph I Histogram
Graph I Pie Chart
Graph I Probability Plot
Graph I Stem-and-Leaf
Graph I Time Series Plot
Manip I Code
Stat I Basic Statistics I Display Descriptive Statistics
Stat I Basic Statistics I Store Descriptive Statistics
Stat I Tables I Tally

This chapter of IPS is concerned with the various ways of presenting and summarizing a data set. By presenting data, we mean convenient and informative methods of conveying the information contained in a data set. There are two basic methods for presenting data, namely graphically and through tabulations. Still, it can be hard to summarize exactly what these presentations are saying about the data. So the chapter also introduces various summary statistics that are commonly used to convey meaningful information in a concise way.

All of these topics can involve much tedious, error prone calculation, if we were to insist on doing them by hand. An important point is that you should almost never rely on hand calculation in carrying out a data analysis. Not only are there many far more important things for you to be thinking about, as the text discusses, but you are also likely to make an error. On the other hand, never blindly trust the computer! Check your results and make sure that they make sense in light of the application. For this, a few simple hand calculations can prove valuable. In working through the problems in IPS, you should try to use Minitab as much as possible, as this will increase your skill with the package and inevitably make your data analyses easier and more effective.

1.1 Tabulating and Summarizing Data

If a variable is categorical, we construct a table using the values of the variable and record the *frequency* (count) of each value in the data and perhaps the *relative frequency* (proportion) of each value in the data as well. These relative frequencies then serve as a convenient summarization of the data.

If the variable is quantitative, we typically *group* the data in some way, i.e., divide the range of the data into nonoverlapping intervals and record the frequency and proportion of values in each interval. Grouping is accomplished using the Manip `I` Code command discussed in I.11.1.

If the values of a variable are *ordered,* we can record the *cumulative distribution,* namely the proportion of values less than or equal to each value. Quantitative variables are always ordered but sometimes categorical variables are as well, e.g., when a categorical variable arises from grouping a quantitative variable.

Often, it is convenient with quantitative variables to record the *empirical distribution function,* which for data values $x_1, \ldots, x_n$ and at a value x is given by

$$\hat{F}(x) = \frac{\#\text{ of } x_i \leq x}{n}$$

i.e., $\hat{F}(x)$ is the proportion of data values less than or equal to x. We can summarize such a presentation via the calculation of a few quantities such as the *first quartile,* the *median,* and the *third quartile* or present the *mean* and the *standard deviation.*

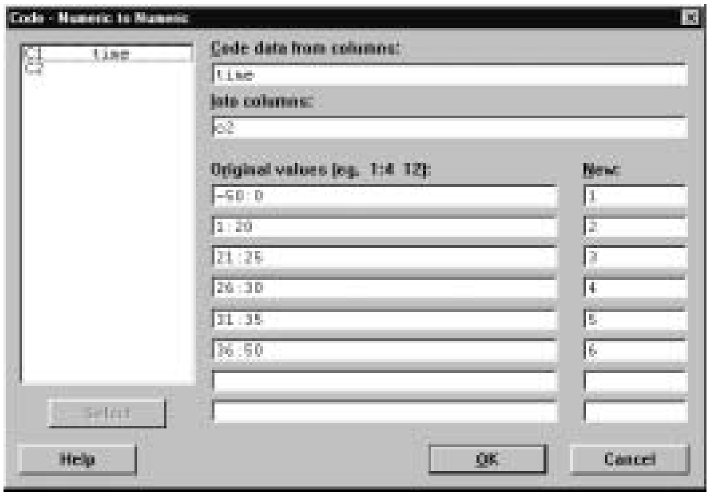
We introduce some new commands to carry out the necessary computations using the data shown in Table 1.1. This is data collected by A.A. Michelson and Simon Newcomb in 1882 concerning the speed of light. We will refer to this hereafter as Newcomb's data and place these in the column C1 with the name `time` in the worksheet called `newcomb`.

28	22	36	26	28	28
26	24	32	30	27	24
33	21	36	32	31	25
24	25	28	36	27	32
34	30	25	26	26	25
-44	23	21	30	33	29
27	29	28	22	26	27
16	31	29	36	32	28
40	19	37	23	32	29
-2	24	25	27	24	16
29	20	28	27	39	23

Table 1.1: Newcomb's data..

1.1.1 Tallying Data

The Stat I Tables I Tally command tabulates categorical data. Consider Newcomb's measurements in Table 1.1. These data range from -44 to 40 (use minimum and maximum in Calc I Calculator to calculate these values). Suppose we decide to group these into the intervals $(-50, 0]$, $(0, 20]$, $(20, 25]$, $(25, 30]$, $(30, 35]$, 0$(35, 40]$. Next we want to record the frequencies, relative frequencies, cumulative frequencies, and cumulative distribution of this grouped variable. First, we used the Manip I Code I Numeric to Numeric command, as described in I.11.1, to recode the data so that every value in $(-50, 0]$ is given the value 1, every value in $(0, 20]$ is given the value 2, etc., and these values are placed in C2. The dialog box for doing this is shown in Display 1.1.



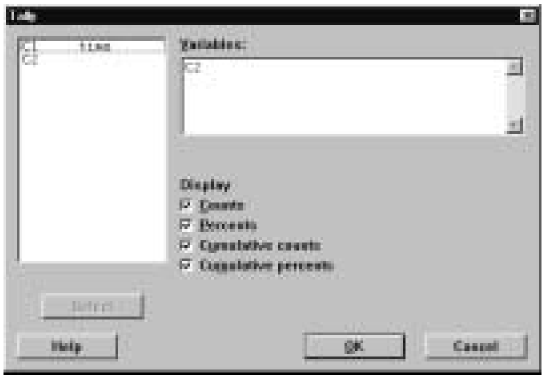
Code data from columns: time

Into columns: c2

Original values (eg. 1:4 12):	New:
-50 : 0	1
1 : 20	2
21 : 25	3
26 : 30	4
31 : 35	5
36 : 50	6

Display 1.1: Dialog box for recoding Newcomb's data.

Next we used the Stat I Tables I Tally command, with the dialog box shown in Display 1.2,



Variables: C2

Display
☑ Counts
☑ Percents
☑ Cumulative counts
☑ Cumulative percents

Display 1.2: Dialog box for tallying the variable C2 in the newcomb worksheet.

to produce the output

```
C2 Count Percent CumCnt CumPct
1      2   3.03       2   3.03
2      4   6.06       6   9.09
3     17   25.76     23  34.85
4     26   39.39     49  74.24
5     10   15.15     59  89.39
6      7   10.61     66 100.00
   N= 66
```

in the Session window.

We can also use the Stat I Tables I Tally command to compute the empirical distribution function of C1 in the newcomb worksheet. First, we must sort the values in C1, from smallest to largest, using the Manip I Sort command described in I.11.6, and then we apply the Stat I Tables I Tally command to this sorted variable.

The general syntax of the corresponding session command **tally** is

tally $E_1 \ldots E_m$

where E_1, ..., E_m are columns of categorical variables, and the command is applied to each column. If no subcommands are given, then only frequencies are computed, while the subcommands **percents** computes relative frequencies, **cumcnts** computes the cumulative frequency function, and **cumpcts** computes the cumulative distribution of C2. Any of the subcommands can be dropped. For example, the commands

```
MTB >sort c1 c3
MTB >tally c3;
SUBC>cumpcnts;
SUBC>store c4 c5.
```

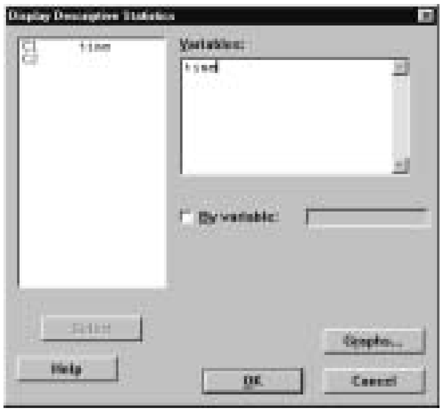
first use the **sort** command to sort the data in C1 from smallest to largest and place the results in C3. The cumulative distribution is computed for the values in C3 with the unique values in C3 stored in C4 and the cumulative distribution at each of the unique values stored in C5 via the **store** subcommand to **tally.**

1.1.2 Describing Data

The Stat I Basic Statistics I Display Descriptive Statistics command is used with quantitative variables to present a numerical summary of the variable values. These values are in a sense a summarization of the empirical distribution of the variable. For example, in the newcomb worksheet the dialog box shown in Display 1.3 leads to the output

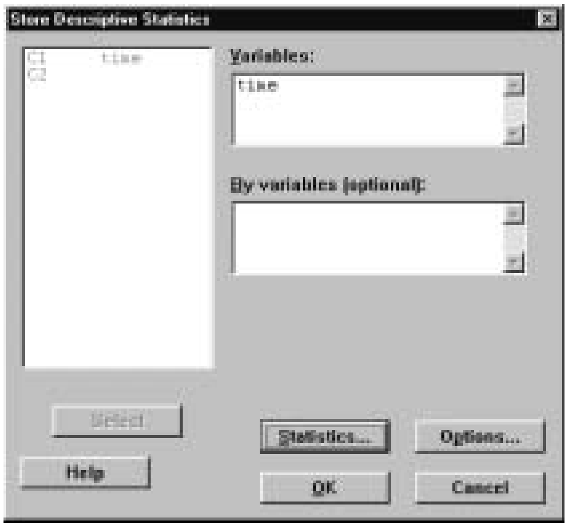
```
Variable  N  Mean  Median TrMean  StDev SE Mean
time     66 26.21   27.00  27.40  10.75    1.32
Variable Minimum Maximum    Q1    Q3
time      -44.00   40.00 24.00 31.00
```

in the Session window. This provides the count N, the mean, median, trimmed mean TrMean (removes lower 5% and upper 5% of the data and averages the rest), standard deviation, standard error of the mean, minimum, maximum, first quartile Q1, and third quartile Q3 of the variable C1. If we want such a summary of a variable by the values of another variable, we check the By variable box and indicate the by variable in the box to the right of this. For example, we might want such a summary for each of the groups we created in II.1.1, and so we would place C2 in this box. Note that a number of summary statistics can also be computed using the Column Statistics discussed in I.10.3.



Display 1.3: Dialog box for computing basic descriptive statistics of a quantitative variable.

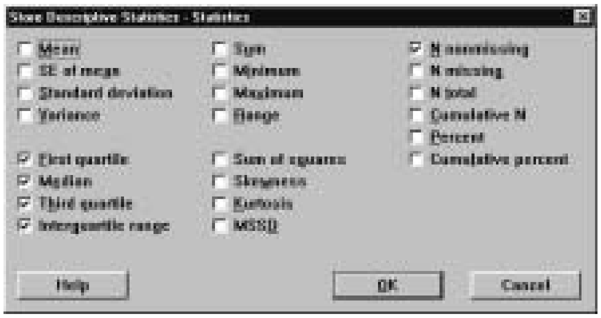
If we wish to compute some basic statistics and store these values for later use, then the Stat I Basic Statistics I Store Descriptive Statistics command is available for this. For example, with the newcomb worksheet this command leads to the dialog box shown in Display 1.4. Clicking on the Statistics button results in the dialog box of Display 1.5 where we have checked First quartile, Median, Third quartile, Interquartile range, and N nonmissing as the statistics we want to compute. The result of these choices is that the next available variables in the worksheet contain these values. So in this case, the values of C3–C7 are as depicted in Display 1.6. Note that these variables are now named as well. Note that many more statistics are available using this command.



Variables:
time

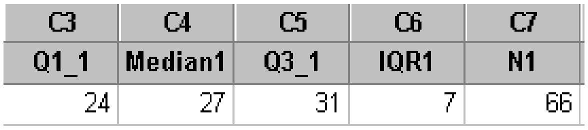
By variables (optional):

Display 1.4: Dialog box for computing and storing various descriptive statistics.



□ Mean
□ SE of mean
□ Standard deviation
□ Variance
☑ First quartile
☑ Median
☑ Third quartile
☑ Interquartile range
□ Sum
□ Minimum
□ Maximum
□ Range
□ Sum of squares
□ Skewness
□ Kurtosis
□ MSSD
☑ N nonmissing
□ N missing
□ N total
□ Cumulative N
□ Percent
□ Cumulative percent

Display 1.5: Dialog box for choosing the descriptive statistics to compute and store.



C3	C4	C5	C6	C7
Q1_1	Median1	Q3_1	IQR1	N1
24	27	31	7	66

Display 1.6: Values obtained for descriptive statistics using dialog boxes in Figures 1.4 and 1.5.

The general syntax of the Session command **describe,** corresponding to Stat I Basic Statistics I Display Descriptive Statistics, is

describe $\mathrm{E}_1 \ldots \mathrm{E}_m$

where E_1, ..., E_m are columns of quantitative variables and the command is applied to each column. A **by** subcommand can also be used. The **stats** command is available in the Session window if we want to store the values of statistics. We refer the reader to **help** for a description of this command.

1.2 Plotting Data in a Graph Window

One of the most informative ways of presenting data is via a plot. There are many different types of plots within Minitab, and which one to use depends on the type of variable you have and what you are trying to learn. In this section we describe how to use the plotting features in Minitab. There are, however, many features of plotting that we will not describe. For example, there are many graphical editing capabilities that allow you to add features, such as titles or legends. Some of these features are accessed via Graph I Layout. We refer the reader to Help for more details on these features.

Each plot in Minitab is made in a *Graph window.* You can make multiple plots and retain each Graph window until you want to delete it simply by clicking the × symbol in the upper right-hand corner. You make any particular Graph window active by clicking in it or by using the Window command. A plot can be saved in an external file in a variety of formats, such as Minitab graph `.mgf`, bitmap `.bmp`, JPEG `.jpg`, etc., using the File I Save Graph As command. If a graph has been saved in the `.mgf` format, it can be reopened using the File I Open Graph command.

1.2.1 Dotplots

The Graph I Dotplot command is used with quantitative variables and produces a plot of each data value as a dot along the x-axis so that you get a general idea of the location of the data and how much scatter there is. Actually, the data is grouped before plotting and multiple observations in a group are stacked over the x-axis. The interval between successive tick (+) marks on the x-axis is divided into 10 equal-length subintervals for the grouping. Typically, one also looks for points that are far from the main scatter of points as these may be identified as *outliers* and, as such, deleted from the data set for subsequent analysis. For example, for the `newcomb` worksheet dialog box in Display 1.7 results in the plot of Display 1.8.

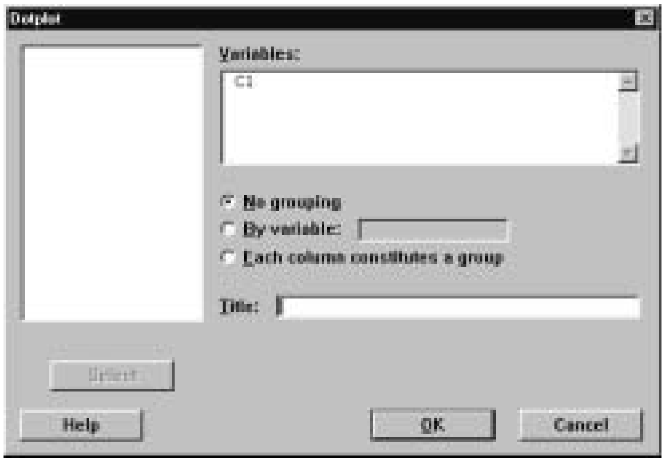
The general syntax of the corresponding Session command **dotplot** is

dotplot $\mathrm{E}_1 \ldots \mathrm{E}_m$

where E_1, ..., E_m are columns, and a dotplot is produced for each. There are a number of subcommands available. The **same** subcommand ensures the scales of the dotplots are the same for each column. The **by** subcommand allows plotting of a variable by the values of another variable with all plots having the same scale. The **increment** subcommand allows for control of the distance between the tick marks and **start** and **end** allow you to specify where the dotplot should begin and end. For example,

```
MTB >dotplot c1;
SUBC>increment=5;
SUBC>start=20 end=35.
```

puts the tick marks 5 units apart, starts the plot at 20, and ends it at 35, so some points are not plotted in this case.

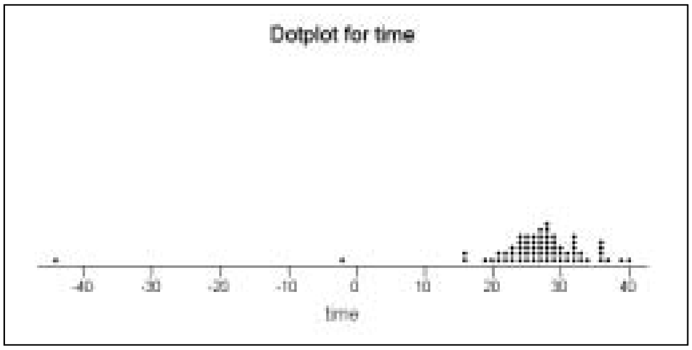


Variables: C1

● No grouping
○ By variable:
○ Each column constitutes a group

Title:

Display 1.7: Dialog box for producing a dotplot.



Display 1.8: Dotplot of the Newcomb data.

1.2.2 Stem-and-Leaf Plots

Stem-and-leaf plots are similar to histograms and are produced by the Graph I Stem-and-Leaf command. These plots are also referred to as *stemplots* as in IPS. For example, using this command with the newcomb worksheet produces the output in the Session window

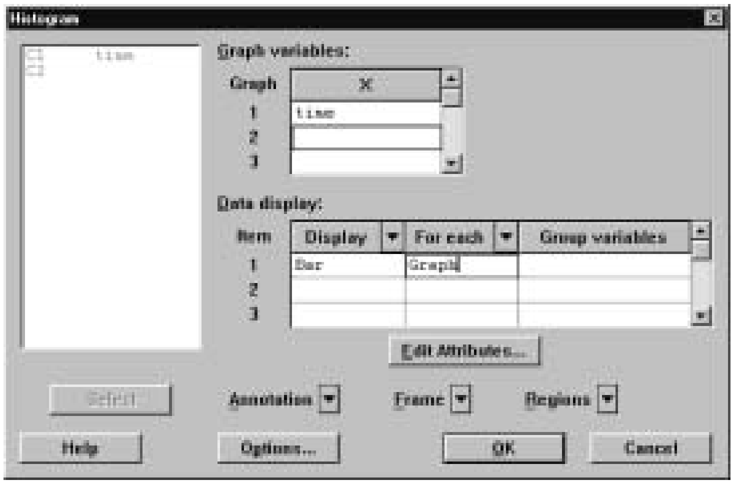
```
Stem-and-leaf of time N = 66
Leaf Unit = 1.0
   1  -4 4
   1  -3
   1  -2
   1  -1
   2  -0 2
   2   0
   5   1 669
 (41)  2 01122333444445555566666777777888888899999
  20   3 0001122222334666679
   1   4 0
```

which is a stem-and-leaf plot of the values in `time`. The first column gives the *depths* for a given stem, i.e., the number of observations on that line and below it or above it, depending on whether or not the observation is below or above the median. The row containing the median is enclosed in parentheses ( ), and the depth is only the observations on that line. If the number of observations is even and the median is the average of values on different rows, then parentheses do not appear. The second column gives the *stems,* as determined by Minitab, and the remaining columns give the ordered *leaves,* where each digit represents one observation. The *Leaf Unit* determines where the decimal place goes after each leaf. So in this example, the first observation is -44.0, while it would be -4.4 if the Leaf Unit were .1. Multiple stem-and-leaf plots can be carried out for a number of columns simultaneously and also for a single variable by the values of another variable.

1.2.3 Histograms

A histogram is a plot where the data are grouped into intervals, and over each such interval a bar is drawn of height equal to the frequency of data values in that interval or of height equal to the relative frequency (proportion) of data values in that interval or of height equal to the *density* of points in that interval, i.e., the proportion of points in the interval divided by the length of the interval. The Graph I Histogram command is used to obtain these plots.

For example, using this command with the `newcomb` worksheet, produces the dialog box shown in Display 1.9. We have placed the variable `time` in the first `X` box to indicate we want a histogram of this variable. We can produce multiple histograms by placing more variables in the `X` boxes. To select the type of histogram to plot, we next click on the Options button, which produces the dialog box of Display 1.10. Here, we have selected a density histogram and have specified the intervals to use for grouping the data by specifying the cutpoints $-45, -30, -15, 0, 15, 30, 45$, which prescribe the intervals $[-45, -30), [-30, -15)$, etc., for the grouping. Alternatively, we could have specified the midpoints of the grouping intervals. The advantage with cutpoints is that subintervals of unequal lengths can be specified. Clicking on the OK buttons in these boxes produces the histogram shown in Display 1.11. As can be seen from the dialog box of Display 1.9, there are a variety of methods for controlling the appearance of the histogram produced, and we refer the reader to the Help button for a description of these.



C1 time
C2

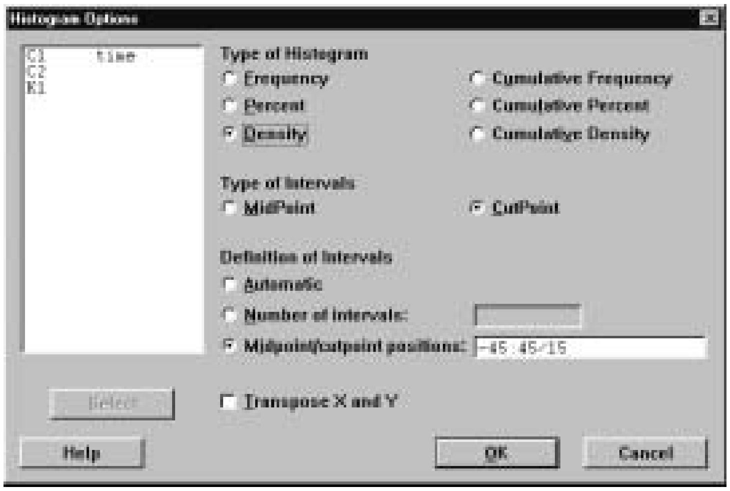
Graph variables:

Graph	X
1	time
2	
3	

Data display:

Item	Display	For each	Group variables
1	Bar	Graph	
2			
3			

Display 1.9: Dialog box for creating a histogram of the time variable in the `newcomb` worksheet.



C1 time
C2
K1

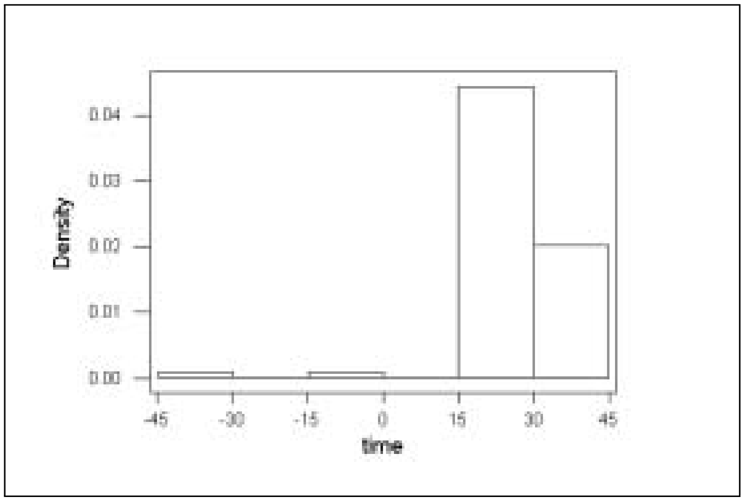
Type of Histogram
○ Frequency ○ Cumulative Frequency
○ Percent ○ Cumulative Percent
● Density ○ Cumulative Density

Type of Intervals
○ MidPoint ● CutPoint

Definition of Intervals
○ Automatic
○ Number of intervals:
● Midpoint/cutpoint positions: -45:45/15

□ Transpose X and Y

Display 1.10: Dialog box for selecting the type of histogram to plot.



Display 1.11: Density histogram of the time variable in the newcomb worksheet.

An important consideration when plotting multiple histograms is to ensure that all the histograms have the same x and y scales so that the plots are visually comparable. This can be accomplished from the dialog box shown in Display 1.9 by Frame I Multiple Graphs and then selecting Same X and same Y.

The session command **histogram** is also available. This has the general syntax

histogram $E_1 \ldots E_m$

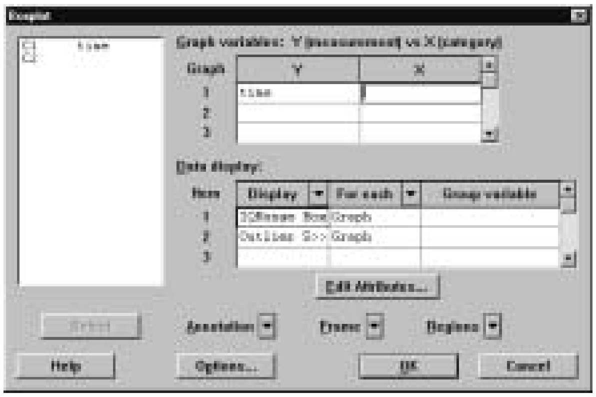
where E_1, ..., E_m correspond to columns. For example, the commands

```
MTB >histogram c1;
SUBC>cutpoints -45 -30 -15 0 15 30 45;
SUBC>density.
```

produce the histogram in Display 1.11 using the **cutpoints** and **density** subcommands. There are also subcommands **midpoints, nintervals,** which specify the number of subintervals, and **frequency** or **percent,** which respectively ensure that the heights of the bar lines equal the frequency and relative frequency of the data values in the interval. Also, the **cumulative** subcommand is available so that the bars represent all the values less than or equal to the endpoint of an interval. The subcommand **same** ensures that multiple histograms all have the same scale.

1.2.4 Boxplots

Boxplots are useful summaries of a quantitative variable and are obtained using the Graph I Boxplot command. Boxplots are used to provide a graphical notion of the location of the data and its scatter in a concise and evocative way. For example, in the newcomb worksheet this command produces the dialog box shown in Display 1.12 and the plot in Display 1.13. The line in the center of the box is the median. The line below the median is the first quartile, also called the *lower hinge,* and the line above is third quartile, also called the *upper hinge.* The difference between the third and first quartile, is called the *interquartile range* or IQR. The vertical lines from the hinges are called *whiskers,* and these run from the hinges to the *adjacent values.* The adjacent values are given by the greatest value less than or equal to the *upper limit* (the third quartile plus 1.5 times the IQR) and by the least value greater than or equal to the *lower limit* (the first quartile minus 1.5 times the IQR). The upper and lower limits are also referred to as the *inner fences.* The *outer fences* are defined by replacing the multiple 1.5 in the definition of the inner fences by 3.0. Values beyond the outer fences are plotted with a * and are called *outliers.* As with the plotting of histograms, multiple boxplots can be plotted for comparison purposes, and again, it is important to make sure that they all have the same scale.



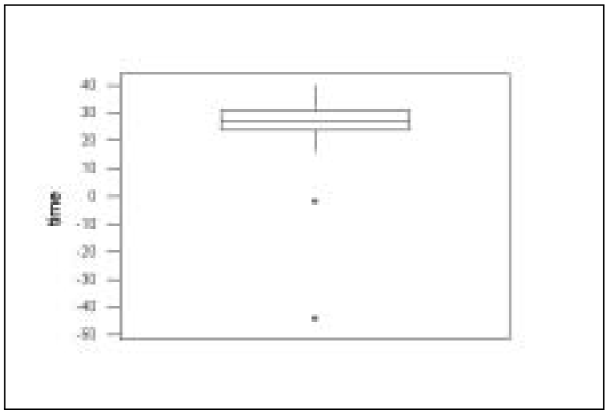
Graph variables: Y (measurement) vs X (category)

Graph	Y	X
1	time	
2		
3		

Data display:

Item	Display	For each	Group variable
1	IQRange Box	Graph	
2	Outlier S>>	Graph	
3			

Display 1.12: Dialog box for producing a boxplot of the time variable in the `newcomb` worksheet.

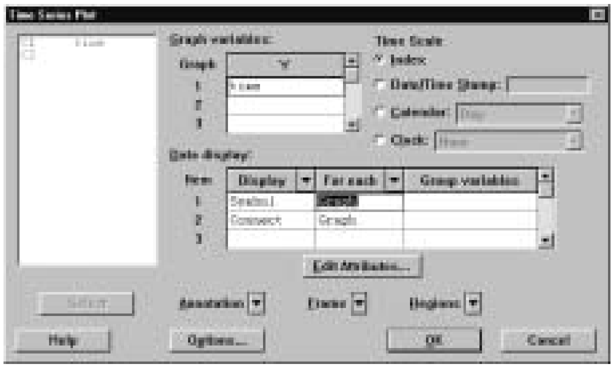


Display 1.13: Boxplot of the time variable in the `newcomb` worksheet.

There is a corresponding session command called **boxplot**. We refer the reader to **help** for more discussion of this command.

1.2.5 Time Series Plots

Often, data are collected sequentially in time. In such a context, it is instructive to plot the values of quantitative variables against time in a time series plot. For this we use the Graph I Time Series Plot command. If we suppose that the data values in time of the newcomb worksheet were obtained in the order they are listed, then applying this command to that data with the dialog box as in Display 1.14 produces the time plot shown in Display 1.15. Notice that in the Data display box we have specified that the graph should plot a symbol for each point and that the symbols plotted should connect via lines. For example, if we had left out connect, only the points would have been plotted. The lines help to visualize the form of the graph. The symbol plotted is a solid circle but other choices could have been made using the Edit Attributes button. Also, for the Time Scale we have chosen Index, which is just the order in which the observations are listed. If these observations were made at periodic time intervals, there are other possible choices that could be more meaningful.



Time Series Plot

Graph variables:

Graph	Y
1	time
2	
3	

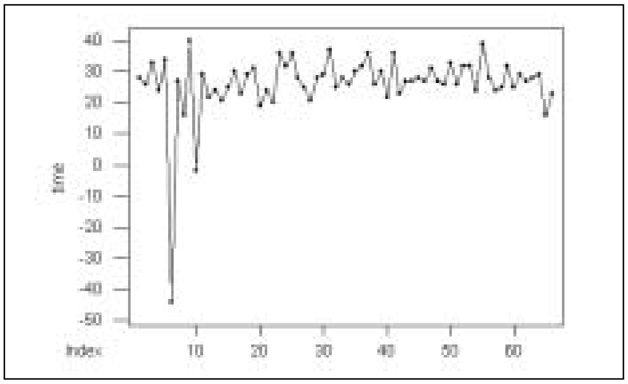
Time Scale

● Index
○ Date/Time Stamp:
○ Calendar: Day
○ Clock: Hour

Data display:

Item	Display	For each	Group variables
1	Symbol	Graph	
2	Connect	Graph	
3			

Display 1.14: Dialog box for a time series plot of the variable time from the newcomb worksheet.

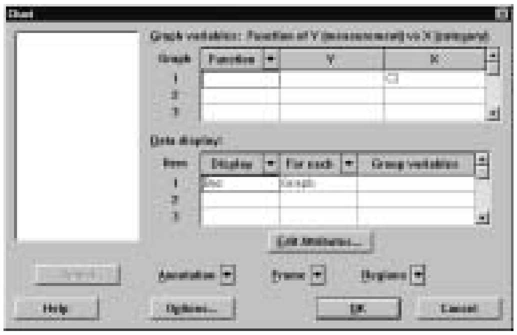


Display 1.15: Time series plot of the variable time from the newcomb worksheet.

There is also a corresponding session command **tsplot**. We refer the reader to **help** for more discussion of this.

1.2.6 Bar Charts

It is also possible to produce various charts using the Graph I Chart command. For example, the dialog box shown in Display 1.16 plots a *bar chart* of the variable C2 in the `newcomb` worksheet. Each distinct value of C1 is plotted along the x-axis simply as a categorical value, not as a quantitative value, and a bar of height equal to the number of times that value occurs in the variable is drawn. A bar chart is a good way to plot categorical variables. There are many possibilities for the types of bar charts drawn, and we refer the reader to the Help button for a discussion of these.



Display 1.16: Dialog box for plotting bar charts.

The corresponding session command is

chart E_1

which produces a bar chart for the values in column E_1.

1.2.7 Pie Charts

A *pie chart* is a disk divided up into wedges where each wedge corresponds to a unique value of a variable, and the area of the wedge is proportional to the relative frequency of the value with which it corresponds. Pie charts can be obtained via Graph I Pie Chart, and there are various features available in the dialog box that can be used to enhance these plots. Pie charts are a common method for plotting categorical variables.

1.3 The Normal Distribution

It is important in statistics to be able to do computations with the normal distribution. The equation of the *density curve* for the normal distribution with mean μ and standard deviation σ is given by

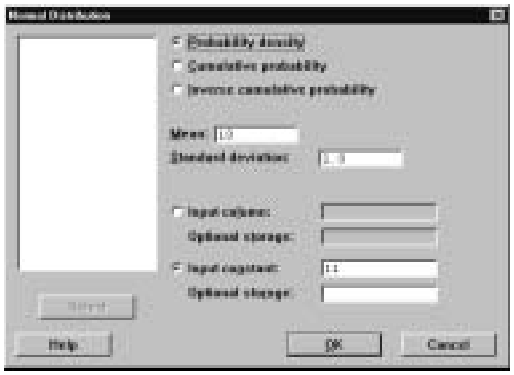
$$\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}$$

where z is a number. We refer to this as the $N(\mu, \sigma)$ density curve. Also of interest is the area under the density curve from $-\infty$ to a number x, i.e., the area between the graph of the $N(\mu, \sigma)$ density curve and the interval $(-\infty, x]$. As noted in IPS, this is a value between 0 and 1. Sometimes, we specify a value p between 0 and 1 and then want to find the point x_p, such that p of the area under the $N(\mu, \sigma)$ density curve lies over $(-\infty, x_p]$. The point x_p is called the *pth percentile* of the $N(\mu, \sigma)$ density curve.

Often, we are given a mean μ and a standard deviation σ and asked to standardize a variable x whose values are in some column, i.e., produce the new variable $z = \frac{x-\mu}{\sigma}$. These arithmetical operations can be carried out using the **let** command as described in I.10.1.

1.3.1 Calculating the Density

Suppose that we want to evaluate the $N(\mu, \sigma)$ density curve at a value x. For this, we use the Calc I Probability Distributions I Normal command. For example, the dialog box in Display 1.17 indicates that we want to evaluate the $N(10, 1)$ density curve at the value $x = 11.0$.



● Probability density
○ Cumulative probability
○ Inverse cumulative probability

Mean: 10
Standard deviation: 1.0

○ Input column:
Optional storage:
● Input constant: 11
Optional storage:

Display 1.17: Dialog box for normal probability calculations.

After clicking on the OK button the output

```
Normal with mean = 10.0000 and standard deviation = 1.00000
    x          f( x )
 11.0000       0.2420
```

is printed in the Session window, which gives the value as .2420. Sometimes, we will want to evaluate the density curve at every value in a column of values, e.g., when we are plotting this curve. For this we simply click on the radio button Input column and type the relevant column in the associated box.

The general syntax of the corresponding session command **pdf** with the **normal** subcommand is

pdf $\mathrm{E}_1 \ldots \mathrm{E}_m$ into $\mathrm{E}_{m+1} \ldots \mathrm{E}_{2m}$;
normal mu $= \mathrm{V}_1$ sigma $= \mathrm{V}_2$.

where E_1, ..., E_m are columns or constants containing numbers and E_{m+1}, ..., E_{2m} are the columns or constants that store the values of the $N(\mu, \sigma)$ density curve at these numbers and $V_1 = \mu$ and $V_2 = \sigma$. If no storage is specified, then the values are printed. For example, if we want to compute the $N(-.5, 1.2)$ density curve at every value between -3 and 3 in increments of .01, the commands

```
MTB >set c1
DATA>-3:3/.01
DATA>end
MTB >pdf c1 c2;
SUBC>normal mu=-.5 sigma=1.2.
```

put the values between -3 and 3 in increments of .01 in C1 using the **set** command. The **pdf** command with the **normal** subcommand calculates the $N(-.5, 1.2)$ density curve at each of these values and puts the outcomes in the corresponding entries of C2. If we plot C2 against C1, we will have a plot of the density curve of this distribution. For this, we use the scatterplot facilities in Minitab as discussed in II.3. Note that with the **normal** subcommand we must also specify the mean and the standard deviation via **mu** and **sigma**.

1.3.2 Calculating the Distribution Function

Suppose that we want to evaluate the area under $N(\mu, \sigma)$ density curve over the interval $(-\infty, x]$. This is the value of the cumulative distribution function of the $N(\mu, \sigma)$ distribution at the value x. For this, we use the Calc I Probability Distributions I Normal as well, but in this case in the dialog box of Display 1.17 we select Cumulative probability instead. Making this change in the dialog box of Display 1.17, we get the output

```
          x       P( X <= x )
    11.0000         0.8413
```

in the Session window. Again, we can evaluate this function at a single point or at every value in a variable.

The general syntax of the corresponding Session command **cdf** command with the **normal** subcommand is

cdf $E_1 \ldots E_m$ into $E_{m+1} \ldots E_{2m}$;
normal mu $= V_1$ sigma $= V_2$.

where E_1, ..., E_m are columns or constants containing numbers and E_{m+1}, ..., E_{2m} are the columns or constants that store the values of the area under $N(\mu, \sigma)$ density curve over the interval from $-\infty$ to these numbers and $V_1 = \mu$ and $V_2 = \sigma$. If no storage is specified, the values are printed.

1.3.3 Calculating the Inverse Distribution Function

Suppose that we want to evaluate percentiles for the $N(\mu, \sigma)$ density curve. Again, we use the Calc I Probability Distributions I Normal command, but in this case, in the dialog box of Display 1.17 we select Inverse cumulative probability instead. Making this change in the dialog box of Display 1.17 and replacing 11 by .75 — recall that the argument to this function must be between 0 and 1 — we get the output

```
P( X <= x )        x
 0.7500        10.6745
```

in the Session window. This indicates that the area to the left of 10.6745 underneath the $N(-.5, 1.2)$ density curve is .75.

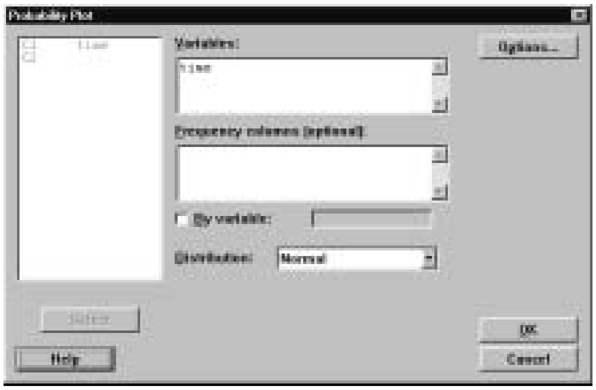
The general syntax of the corresponding session command **invcdf** with the **normal** subcommand is

invcdf $\text{E}_1 \ldots \text{E}_m$ into $\text{E}_{m+1} \ldots \text{E}_{2m}$;
normal mu $= \text{V}_1$ sigma $= \text{V}_2$.

where E_1, ..., E_m are columns or constants containing numbers between 0 and 1 and E_{m+1}, ..., E_{2m} are the columns or constants that store the values of the percentiles of the $N(\mu, \sigma)$ density curve at these numbers and where $\text{V}_1 = \mu$ and $\text{V}_2 = \sigma$. If no storage is specified, then the values are printed.

1.3.4 Normal Probability Plots

Some statistical procedures require that we assume that values for some variables are a sample from a normal distribution. A *normal probability plot* is a *diagnostic* that checks for the reasonableness of this assumption. To create such a plot, we use the Graph I Probability Plot command. For example, using this command on the newcomb worksheet we get the dialog box in Display 1.18 where we have placed time in the Variables box. Clicking on the OK button produces the plot in Display 1.19. The normal probability plot is given by the dark dotted curve. The plot also contains other information and further output is printed in the Session window. Of course, the plot should be like a straight line and it is not in this case.



Probability Plot

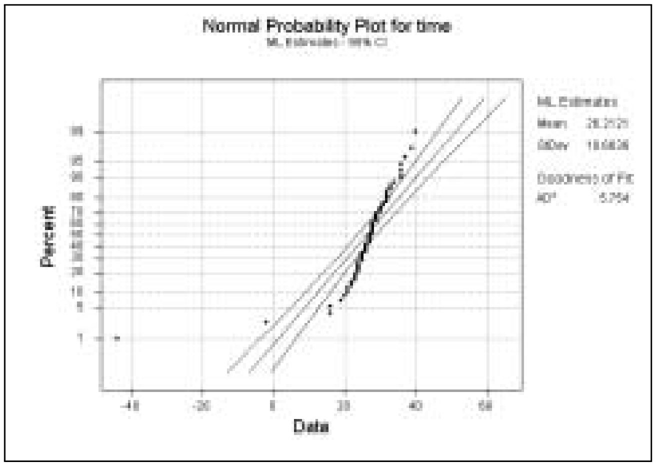
Variables: time

Frequency columns (optional):

☐ By variable:

Distribution: Normal

Display 1.18: Dialog box for producing normal probability plots.



Display 1.19: Normal probability plot of the time variable in the newcomb worksheet.

The session commands

```
MTB >nscores c1 c3
MTB >plot c3*c1
```

produce a normal probability plot like that shown in Display 2.3. The **plot** command will be discussed much more extensively in II.3. The **nscores** (*normal scores*) command relies on some concepts that are beyond the level of this course so we do not discuss this further.

1.4 Exercises

When the data for an exercise come from an exercise in IPS, the IPS exercise number is given in parentheses ( ). All computations in these exercises are to be carried out using Minitab, and the exercises are designed to ensure that you have a reasonable understanding of the Minitab material in this chapter. Generally, you should be using Minitab to do all the computations and plotting required for the problems in IPS.

1. Using Newcomb's measurements in Table 1.1, create a new variable by grouping these values into three subintervals [−50, 0), [0, 20), [20, 50). Calculate the frequency distribution, the relative frequency distribution, and the cumulative distribution of this ordered categorical variable.

2. (1.21) Use Minitab to print the empirical distribution function. From this, determine the first quartile, median, and third quartile. Also, use the empirical distribution function to compute the 10th and 90th percentiles.

3. Use Minitab to produce the stemplot of Example 1.4 of IPS.

4. Use Minitab to produce the time plot of Example 1.5 of IPS.

5. (1.29) Use Minitab commands for the stemplot and the time plot. Use Minitab commands to compute a numerical summary of this data, and justify your choices.

6. (1.30) Transform the data in this problem by subtracting 5 from each value and multiplying by 10. Calculate the means and standard deviations, using any Minitab commands, of both the original and transformed data. Compute the ratio of the standard deviation of the transformed data to the standard deviation of the original data. Comment on this value.

7. (1.30) Transform this data by multiplying each value by 3. Compute the ratio of the standard deviation to the mean (called the *coefficient of variation*) for the original data and for the transformed data. Justify the outcome.

8. For the $N(6, 1.1)$ density curve, compute the area between the interval $(3, 5)$ and the density curve. What number has 53% of the area to the left of it for this density curve?

9. Use Minitab commands to verify the 68-95-99.7 rule for the $N(2, 3)$ density curve.

10. Calculate and store the values of the $N(0, 1)$ density curve at each value in $[-3, 3]$ using an increment of .01. Put the values in the interval $[-3, 3]$ in C1 and the values of the density curve in C2. Using the command `plot C2*C1`, plot the density curve. Comment on the shape of this curve.

11. Use Minitab commands to make the normal quantile plots presented in Figures 1.31 and 1.32 of IPS.

Chapter 2

Looking at Data – Relationships

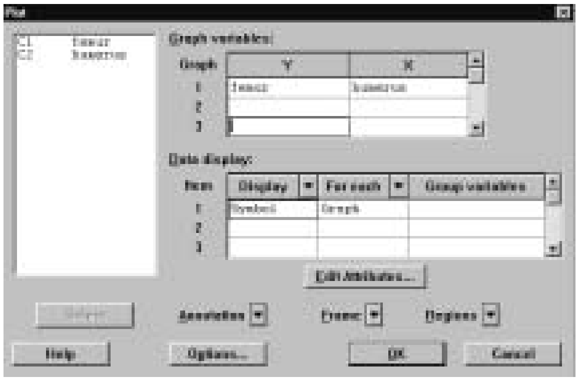
New Minitab commands discussed in this chapter

Graph I Plot
Stat I Basic Statistics I Correlation
Stat I Regression I Fitted Line Plot
Stat I Regression I Regression

In this chapter, Minitab commands are described that permit the analysis of relationships among two variables. The methods are different depending on whether or not both variables are quantitative, both variables are categorical, or one is quantitative and the other is categorical. This chapter considers relationships between two quantitative variables with the remaining cases discussed in later chapters. Graphical methods are very useful in looking for relationships among variables, and we examine various plots for this.

2.1 Scatterplots

A *scatterplot* of two quantitative variables is a useful technique when looking for a relationship between two variables. By a scatterplot we mean a plot of one variable on the y-axis against the other variable on the x-axis. For example, consider Example 2.4 in IPS, where we are concerned with the relationship between the length of the femur and the length of the humerus in an extinct species. Suppose that we have input the data so that length of the femur measurements are in C1, which has been named `femur`, and the length of the humerus measurements are in C2, which has been named `humerus`, of the worksheet `archaeopteryx`. The command Graph I Plot produces the dialog box of Display 2.1, where we have placed `femur` into the first box for the y variable and `humerus` in the first box for the x variable. This produces the plot shown in Display 2.2. Note that we could alter the plotting symbol using the dialog box that appears when we click on the Edit Attributes box. Using the dialog box that appears when you click on the Annotation button, it is possible to give the plot a title, label plotted points, etc. Using the dialog box that appears when you click on the Frame button, you can change the labels on the axes. Rather than just plotting the points in a scatterplot, you can add *connection lines* (join the points with lines), add *projection lines* (drop a line from each point to the x-axis), and add *areas* (fill in the area under a polygon joining the points). Also, you can employ the scatterplot smoother *lowess* to plot a piecewise linear continuous curve through the scatter of points. These features are available via Graph I Plot I Display. There are a number of other features that allow you to control the appearance of the plot.



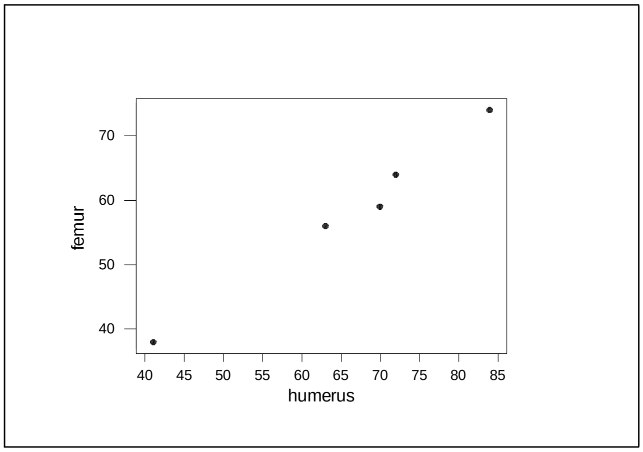
Graph variables:

Graph	Y	X
1	femur	humerus
2		
3		

Data display:

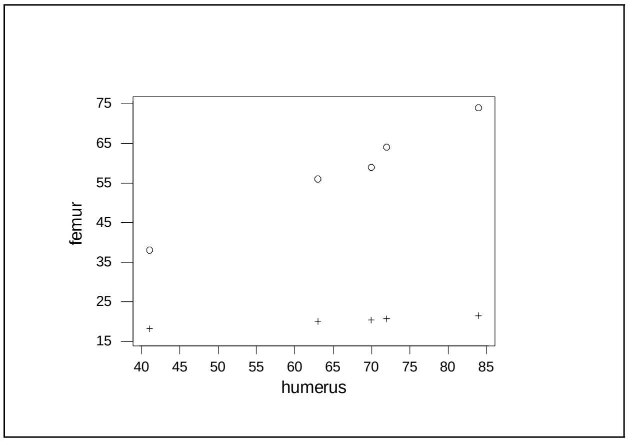
Item	Display	For each	Group variables
1	Symbol	Graph	
2			
3			

Display 2.1: Dialog box for producing a scatterplot.



Display 2.2: Scatter plot of femur length (C1) versus humerus length (C2) of Example 2.4 in IPS.

It is also possible to have multiple scatterplots on the same plot. For example, suppose that C3 in the `archaeopteryx` worksheet contains the natural log of the `femur` variable. We obtained the plot of Display 2.3 by adding another pair of variables to the second Graph variables box as in Display 2.1 with C3 as the y variable and `humerus` as the x variable. To put these scatterplots on the same plot use Frame `I` Multiple Graphs and click on the Overlay graphs on the same page radio button.



Display 2.3: Multiple scatterplots in the same plot.

The technique of *brushing* is available after obtaining the plot to see which observations (rows) the points correspond to. This is helpful in identifying the points that correspond to outliers. Brushing is accessed from the toolbar just below the menu bar by clicking on the brush when the Graph window is active.

The corresponding session command is **plot.** For example,

```
MTB > plot femur*humerus
```

produces the plot of Display 2.2. Note that the first variable is plotted along the y-axis, and the second variable is plotted along the x-axis. There are various subcommands that can be used with **plot,** and we refer the reader to Help for a description of these.

There are a number of additional plots available in Minitab that are related to the scatterplot. For example, a *marginal plot* of two variables is a scatterplot of one variable against the other where in addition histograms, dotplots or boxplots are plotted along the sides of the scatterplot for each variable. These are available via the menu command Graph `I` Marginal Plot. *Draftsman plots* allow you to produce a number of scatterplots in a rectangular array so that they can be compared. For example, you may want to plot C1 against C3, C2 against C3, C1 against C4, and C2 against C4 and see all of these in a common plot. This capability is available via the menu command Graph `I` Draftsman Plot and filling in the dialog box. *Matrix plots* provide a mechanism for placing a number of scatterplots in a rectangular array or matrix so that they can be directly compared or examined for relationships. Matrix plots are available via the command Graph I Matrix Plot. Also three-dimensional scatterplots are available via Graph I 3D Plot and contour plots via Graph I Contour Plot.

2.2 Correlations

While a scatterplot is a convenient graphical method for assessing whether or not there is any relationship between two variables, we would also like to assess this numerically. The *correlation coefficient* provides a numerical summarization of the degree to which a linear relationship exists between two quantitative variables, and this can be calculated using the Stat I Basic Statistics I Correlation command. For example, applying this command to the `femur` and `humerus` variables of the worksheet `archaeopteryx`, i.e., the data of Example 2.4 in IPS and depicted in Display 2.2, we obtain the output

```
Pearson correlation of femur and humerus = 0.994
P-Value = 0.001
```

in the Session window. For now, we ignore the number recorded as `P-Value.`

The general syntax of the corresponding session command **correlate** is given by

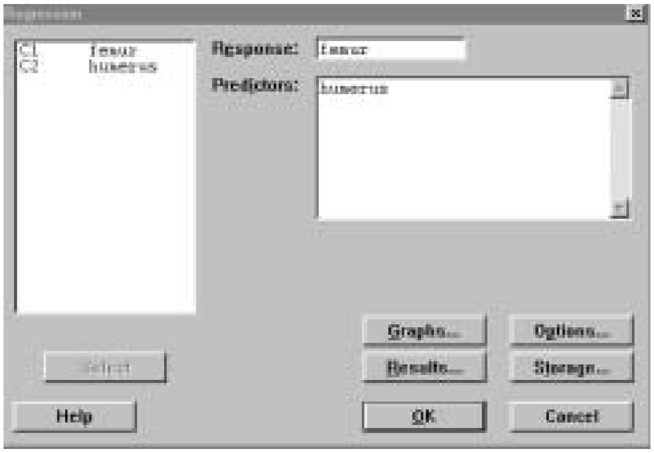
correlate $E_1 \ldots E_m$

where E_1, ..., E_m are columns corresponding to numerical variables, and a correlation coefficient is computed between each pair. This gives $m(m-1)/2$ correlation coefficients. The subcommand **nopvalues** is available if you want to suppress the printing of P-values.

2.3 Regression

Regression is another technique for assessing the strength of a linear relationship existing between two variables and it is closely related to correlation. For this, we use the Stat I Regression command.

As noted in IPS, the regression analysis of two quantitative variables involves computing the least-squares line $y = a + bx,$ where one variable is taken to be the response variable y and the other is taken to be the explanatory variable x. Note that the least squares line is different depending upon which choice is made. For example, for the data of Example 2.4 in IPS and plotted in Display 2.2 letting `femur` be the response and `humerus` be the predictor or explanatory variable, the Stat I Regression I Regression command leads to the dialog box of Display 2.4, where we have made the appropriate entries in the Response and Predictors boxes. Clicking on the OK button leads to the output of Display 2.5 being printed in the Session window. This gives the least-squares line as $y = 3.70 + .826x$, i.e., $a = 3.70$ and $b = .826$, which we also see under the `Coef` column in the first table. In addition, we obtain the value of the square of the correlation coefficient, also known as the *coefficient of determination,* as `R-Sq = 98.8%`. We will discuss the remaining output from this command in II.10.

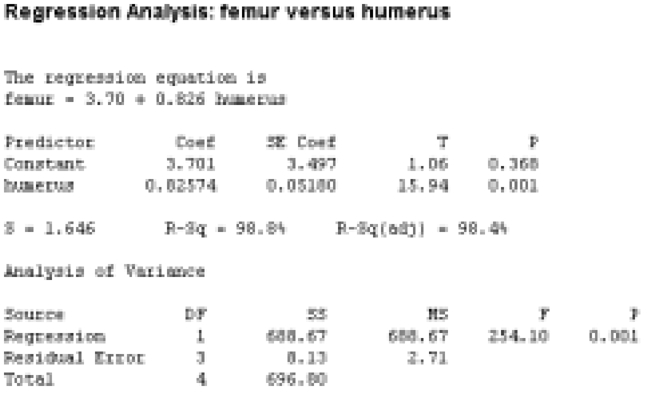


C1 femur
C2 humerus

Response: femur

Predictors: humerus

Display 2.4: Dialog box for a regression analysis.

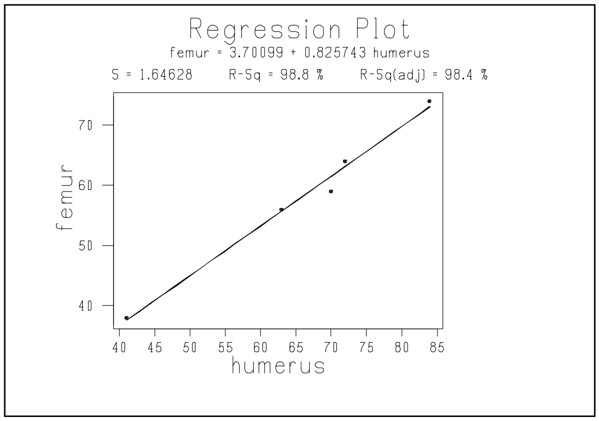


```
Regression Analysis: femur versus humerus


The regression equation is
femur = 3.70 + 0.826 humerus

Predictor        Coef     SE Coef          T        P
Constant        3.701       3.497       1.06    0.368
humerus       0.82574     0.05180      15.94    0.001

S = 1.646       R-Sq = 98.8%     R-Sq(adj) = 98.4%

Analysis of Variance

Source            DF          SS          MS        F        P
Regression         1      688.67      688.67   254.10    0.001
Residual Error     3        8.13        2.71
Total              4      696.80
```

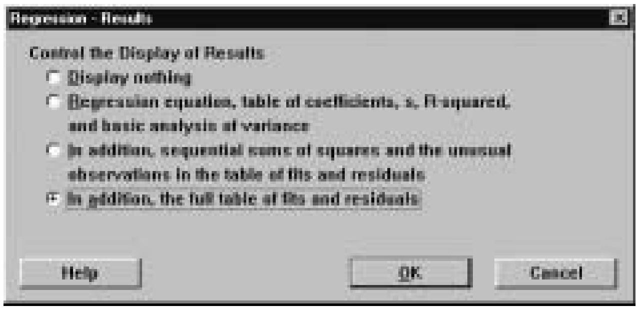
Display 2.5: Output from the dialog box of Display 2.4.

It is very convenient to have a scatterplot of the points together with the least-squares line. This can be accomplished using the Stat I Regression I Fitted Line Plot command. Filling in the dialog box for this command as in Display 2.4 produces the output in the Session window of Display 2.5 together with the plot of Display 2.6.

There are some additional quantities that are often of interest in a regression analysis. For example, you may wish to have the fitted values $\hat{y} = a + bx$ at each x value printed as well as the residuals $y - \hat{y}$. Clicking on the Results button in the dialog box of Display 2.4 and filling in the ensuing dialog box as in Display 2.7 results in these quantities being printed in the Session window as well as the output of Display 2.5.



Display 2.6: Scatterplot of femur versus humerus in the archaeopteryx worksheet together with the least-squares line.

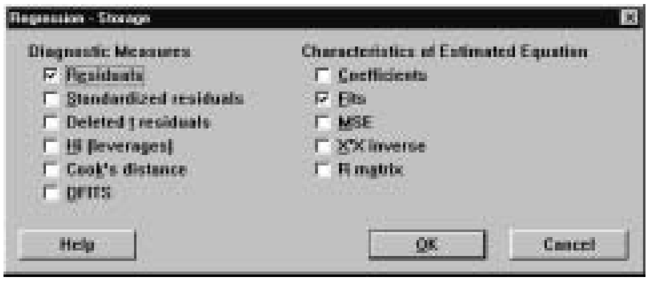

Regression - Results

Control the Display of Results

- ○ Display nothing
- ○ Regression equation, table of coefficients, s, R-squared, and basic analysis of variance
- ○ In addition, sequential sums of squares and the unusual observations in the table of fits and residuals
- ● In addition, the full table of fits and residuals

Display 2.7: Dialog box for controlling output for a regression analysis.

You will probably want to keep these values for later work. In this case, clicking on the Storage button of Display 2.4 and filling in the ensuing dialog box as in Display 2.8 results in these quantities being saved in the next two available columns — in this case, C3 and C4 — with the names `resl1` and `fits1` for the residuals and fits, respectively.


Regression - Storage

Diagnostic Measures

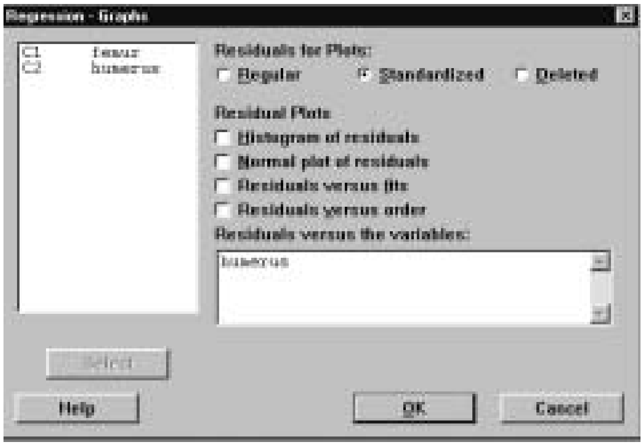
- ☑ Residuals
- ☐ Standardized residuals
- ☐ Deleted t residuals
- ☐ Hi (leverages)
- ☐ Cook's distance
- ☐ DFITS

Characteristics of Estimated Equation

- ☐ Coefficients
- ☑ Fits
- ☐ MSE
- ☐ X'X inverse
- ☐ R matrix

Display 2.8: Dialog box for storing various quantities computed as part of a regression analysis.

Even more likely is that you will want to plot the residuals as part of assessing whether or not the assumptions that underlie a regression analysis make sense in the particular application. For this, click on the Graphs button in the dialog box of Display 2.4. The dialog box of Display 2.9 becomes available. Notice that we have requested that the *standardized residuals* — each residual divided by its standard error — be plotted, and this plot appears in Display 2.10. All the standardized residuals should be in the interval $(-3, 3)$, and no pattern should be discernible. In this case, this residual plot looks fine. From the dialog box of Display 2.9, we see that there are many other possibilities for residual plots.



Regression - Graphs

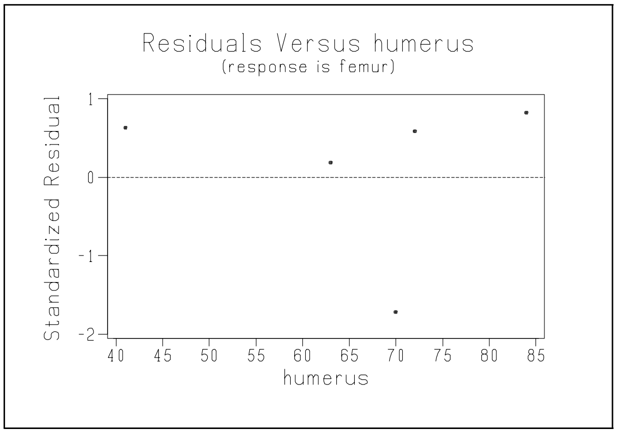
C1 femur
C2 humerus

Residuals for Plots:
○ Regular ● Standardized ○ Deleted

Residual Plots
□ Histogram of residuals
□ Normal plot of residuals
□ Residuals versus fits
□ Residuals versus order

Residuals versus the variables:
humerus

Display 2.9: Dialog box for selecting various residual plots as part of a regression analysis.



Display 2.10: Plot of the standardized residuals versus humerus after regressing femur against humerus in the archaeopteryx worksheet.

The corresponding session command is given by **regress,** and by using the subcommands **pfits, residual,** and **sresidual** we can calculate and store *fitted values, residuals,* and *standardized residuals,* respectively. For example,

```
MTB > regress c1 1 c2;
SUBC> fits c3;
SUBC> residuals c4;
SUBC> sresiduals c5.
```

gives the output of Display 2.5 and also stores the fitted values in C3, stores the residuals $y - \hat{y}$ in C4, and stores the standardized residuals in C5. Note that the 1 in `regress c1 1 c2` refers to the number of predictors we are using to predict the response variable. To plot the standardized residuals against `humerus,` we use

```
MTB > plot c5*c2
```

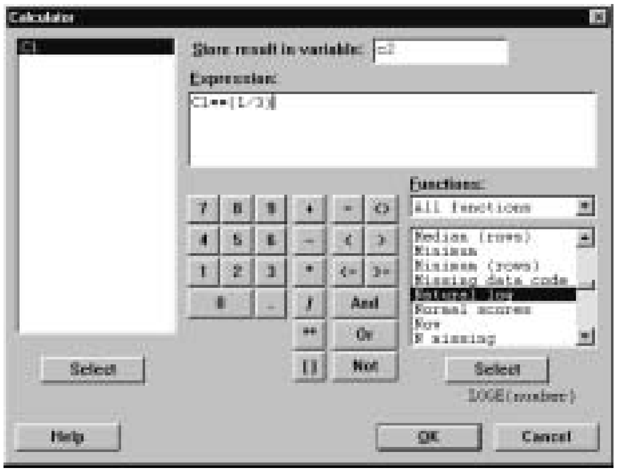
which results in a plot like Display 2.10 but with different labels on the x axis.

2.4 Transformations

Sometimes, transformations of the variables are appropriate before we carry out a regression analysis. This is accomplished in Minitab using the Calc I Calculator command and the arithmetical and mathematical operations discussed in I.10.1 and I.10.2. In particular, when a residual plot looks bad, sometimes this can be fixed by transforming one or more of the variables using a simple transformation, such as replacing the response variable by its logarithm or something else. For example, if we want to calculate the cube root — i.e., $x^{1/3}$ — of every value in C1 and place these in C2, we use the Calc I Calculator command and the dialog box as depicted in Display 2.11. Alternatively, we could use the session command **let** as in

```
MTB > let c2=c1**(1/3)
```

which produces the same result.



Store result in variable: c2

Expression: C1**(1/3)

Display 2.11: Dialog box for calculating transformations of variables.

2.5 Exercises

When the data for an exercise come from an exercise in IPS, the IPS exercise number is given in parentheses ( ). All computations in these exercises are to be carried out using Minitab, and the exercises are designed to ensure that you have a reasonable understanding of the Minitab material in this chapter. Generally, you should be using Minitab to do all the computations and plotting required for the problems in IPS.

1. (2.10) Calculate the least-squares line and make a scatterplot of Fuel used against Speed together with the least-squares line. Plot the standardized residuals against Speed. What is the squared correlation coefficient between these variables?

2. (2.11) Make a scatterplot of Rate against Mass where the points for different Sexes are labeled differently (use Minitab for the labeling, too) and with the least-squares line on it. Hint: Make use of the stack command discussed in I.11.7.

3. Place the values 1 through 100 with an increment of .1 in C1 and the square of these values in C2. Calculate the correlation coefficient between C1 and C2. Multiply each value in C1 by 10, add 5, and place the results in C3. Calculate the correlation coefficient between C2 and C3. Why are these correlation coefficients the same?

4. Place the values 1 through 100 with an increment of .1 in C1 and the square of these values in C2. Calculate the least-squares line with C2 as response and C1 as explanatory variable. Plot the standardized residuals. If you see such a pattern of residuals what transformation, might you use to remedy the problem?

5. (2.54) For the data in this problem, numerically verify the algebraic relationship that exists between the correlation coefficient and the slope of the least-squares line.

6. For Example 2.17 in IPS, calculate the least-squares line and reproduce Display 2.21. Calculate the sum of the residuals and the sum of the squared residuals and divide this by the number of data points minus 2. Is there anything you can say about what these quantities are equal to in general?

7. (2.62) Use Minitab to do all the calculations in this problem.

8. Place the values 1 through 10 with an increment of .1 in C1, and place $\exp(-1 + 2x)$ of these values in C2. Calculate the least-squares line using C2 as the response variable, and plot the standardized residuals against C1. What transformation would you use to remedy this residual plot? What is the least-squares line when you carry out this transformation?

Chapter 3

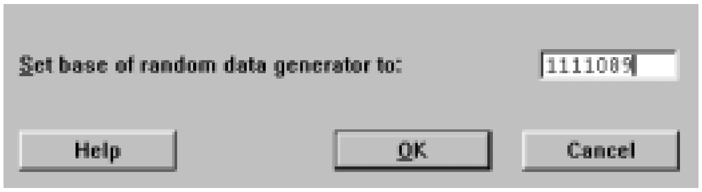
Producing Data

New Minitab commands discussed in this chapter

Calc I Set Base
Calc I Random Data

This chapter is concerned with the collection of data, perhaps the most important step in a statistical problem, as this determines the quality of whatever conclusions are subsequently drawn. A poor analysis can be fixed if the data collected are good by simply redoing the analysis. But if the data have not been appropriately collected, then no amount of analysis can rescue the study. We discuss Minitab commands that enable you to generate samples from populations and also to randomly allocate treatments to experimental units.

Minitab uses computer algorithms to mimic randomness. Still, the results are not truly random. In fact, any simulation in Minitab can be repeated, with exactly the same results being obtained, using the Calc I Set Base command. For example, in the dialog box of Display 3.1 we have specified the base, or seed, random number as 1111089. The base can be any integer. When you want to repeat the simulation, you give this command, with the same integer. Provided you use the same simulation commands, you will get the same results. This can also be accomplished using the session command **base** V, where V is an integer.



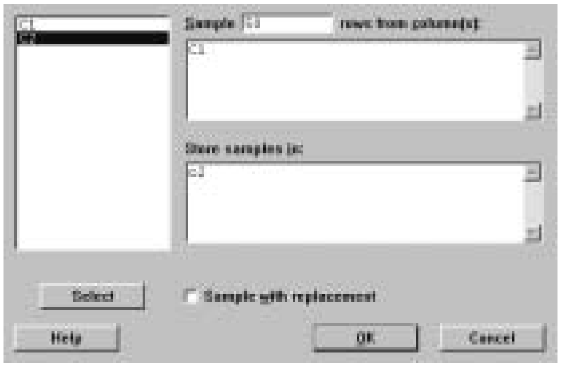
Set base of random data generator to: 1111089

Display 3.1: Dialog box for setting base or seed random number.

3.1 Generating a Random Sample

Suppose that we have a large population of size N and we want to select a sample of $n < N$ from the population. Further, we suppose that the elements of the population are ordered, i.e., we have been able to assign a unique number $1, \ldots, N$ to each element of the population. To avoid selection biases, we want this to be a *random sample,* i.e., every subset of size n from the population has the same "chance" of being selected. As discussed in IPS, this implies that we generate our sample so that every subset of size n in the population has the same chance of being chosen. We can do this physically by using some simple random system, such as chips in a bowl or coin tossing. We could also use a table of random numbers, or, more conveniently, we can use computer algorithms that mimic the behavior of random systems.

For example, suppose there are 1000 elements in a population, and we want to generate a sample of 50 from this population without replacement. We can use the Calc I Random Data I Sample from Columns command to do this. For example, suppose we have labeled each element of the population with a unique number in $1, 2, \ldots, 1000$, and, further, we have put these numbers in C1 of a worksheet. The dialog box of Display 3.2 results in a random sample of 50 being generated without replacement from C1 and stored in C2.



Sample [illegible] rows from column(s): C1

Store samples in: C2

□ Sample with replacement

Display 3.2: Dialog box for generating a random sample without replacement.

Printing this sample gives the output

```
MTB > print c2
C2
 441 956  87 736 185 515 883 957 690
 438 205 760 246  16 321 371 493 393
 538 348  70  54 362 492 182 841 287
 277 112 610 890 503 332 413 886 798
 764 584 566 495 547 488 206 557 263
 414 613 618 685 864
```

in the Session window. So now we go to the population and select the elements labeled 441, 956, 87, etc. The algorithm that underlies this command is such that we can be confident that this sample of 50 is like a random sample.

The general syntax of the corresponding session command **sample** is

sample V $E_1 \ldots E_m$ put into $E_{m+1} \ldots E_{2m}$

where V is the sample size n and V rows are sampled from the columns E_1, ..., E_m and stored in columns E_{m+1}, ..., E_{2m}. If we wanted to sample with replacement — i.e., after a unit is sampled, it is placed back in the population so that it can possibly be sampled again — we use the **replace** subcommand. Of course, for simple random sampling, we do not use the **replace** subcommand. Note that the columns can be numeric or text.

Sometimes we want to generate *random permutations,* i.e., $n = N,$ and we are simply reordering the elements of the population. For example, in experimental design, suppose we have $N = n_1 + \cdots + n_k$ experimental units and k treatments, and we want to allocate n_i applications of treatment i. Suppose further that we want all possible such applications to be equally likely. Then we generate a random permutation $(l_1, \ldots, l_N)$ of $(1, \ldots, N)$ and allocate treatment 1 to those experimental units labeled $l_1, \ldots, l_{n_1}$, allocate treatment 2 to those experimental units labeled $l_{n_1+1}, \ldots, l_{n_1+n_2}$, etc. For example, if we have 30 experimental units and 3 treatments and we want to allocate 10 experimental units to each treatment, placing the numbers $1, 2, \ldots, 30$ in C1 and using the Calc I Random Data I Sample from Columns command as in the dialog box of Display 3.2, but with 30 in the Sample box, generates a random permutation of $1, 2, \ldots, 30$ in C2. Implementing this gives us the random permutation

```
MTB > print c2
C2
 13  7 26  8 22 23 28 17  3 25
 9   2 14 29 15 18  6 11 16  5
 12 27  4 30 20 24  1 19 21 10
```

and for the treatment allocation you can read the numbers row-wise or column-wise, as long as you are consistent. Row-wise is probably best, as this is how the numbers are stored in C2, and so you can always refer back to C2 (presuming you save your worksheet) if you get mixed up.

The above examples show how to directly generate a sample from a population of modest size. But what happens if the population is huge or it is not convenient to label each unit with a number? For example, suppose we have a population of size 100,000 for which we have an ordered list and we want a sample of size 100. In this case more sophisticated techniques need to be used, but simple random sampling can still typically be accomplished (see Exercise 3.3 for a simple method that works in some contexts).

Simple random sampling corresponds to sampling without replacement, i.e., after we randomly select an element from the population, we do not return it to the population before selecting the next sample element. Sampling with replacement corresponds to replacing each sample element in the population after selecting it and recording only the element that was obtained. So at each selection, every element has the same chance of being selected, and an element may appear more than once in the sample. Notice that we can also sample with replacement if we check the Sample with replacement box in the dialog box of Display 3.2.

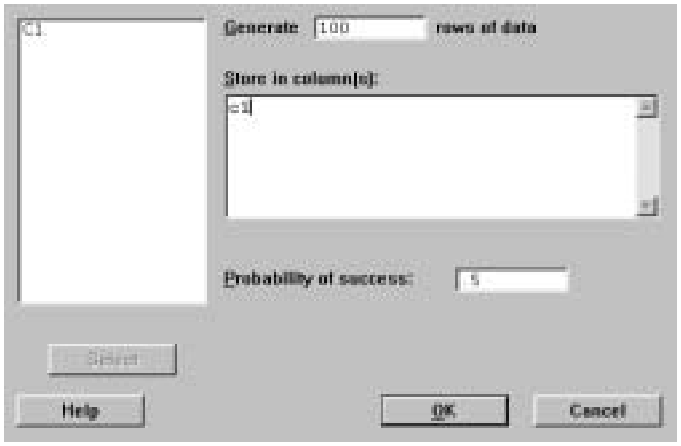
3.2 Sampling from Distributions

Once we have generated a sample from a population, we measure various attributes of the sampled elements. For example, if we were sampling from a population of humans, we might measure each sampled unit's height. The height for the sample unit is now a random variable that follows the height distribution in the population from which we are sampling. For example, if 80% of the people in the population are between 4.5 feet and 6 feet, then under *repeated sampling* of an element from the population (with replacement) in the long run, 80% of the sampled units will have their heights in this range.

Sometimes, we want to sample directly from this population distribution, i.e., generate a number in such a way that under repeated sampling in the long run the proportion of values falling in any range agrees with that prescribed by the population distribution. Of course, we typically don't know the population distribution, as this is what we want to find out about in a statistical investigation. Still, there are many instances where we want to pretend that we do know it and simulate from this distribution, e.g., perhaps we want to consider the effect of various choices of population distribution on the sampling distribution of some statistic of interest.

There are computer algorithms that allow us to do this for a variety of distributions. In Minitab, this is accomplished using the Calc I Random Data command. For example, suppose that we want to simulate the tossing of a fair coin (a coin where head and tail are equally likely as outcomes). The Calc I Random Data I Bernoulli command together with the dialog box of Display 3.3 generates a sample of 100 from the $Bernoulli(.5)$ distribution and places these values in C1. A random variable has a $Bernoulli(p)$ distribution if the probability the variable equals 1 — success — is p and the probability the variable equals 0 — failure — is $1-p$. So to generate a sample of n from the $Bernoulli(p)$ distribution, we put n in the Generate box and p in the Probability of success box. In such a case, we are simulating the tossing of a coin that produces a head on a single toss with probability p, i.e., the long-run proportion of heads that we observe in repeated tossing is p. Note that we can generate m samples of size n by putting m distinct columns in the Store in column(s) box.

Often, a normal distribution with some particular mean and standard deviation is considered a reasonable assumption for the distribution of a measurement in a population. For example, the Calc I Random Data I Normal command together with the dialog box of Display 3.4 generates a sample of 200 from the $N(5.2, 1.3)$ distribution and places this sample in C1. To generate a sample of n from the $N(\mu, \sigma)$ distribution, we put n in the Generate box, μ in the Mean box, and σ in the Standard deviation box.



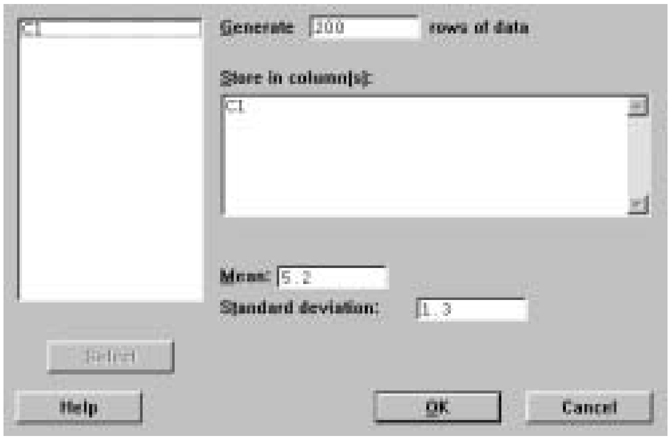
Generate 100 rows of data

Store in column(s):

c1

Probability of success: .5

Display 3.3: Dialog box for generating a sample of 100 from the $Bernoulli(.5)$ distribution.



Generate 200 rows of data

Store in column(s):

C1

Mean: 5.2

Standard deviation: 1.3

Display 3.4: Dialog box for generating a sample of 200 from a $N(5.2, 1.3)$ distribution.

The general syntax of the corresponding session command **random** is

random V into $\mathrm{E}_1 \ldots \mathrm{E}_m$

and this puts a sample of size V into each of the columns E_1, ..., E_m, according to the distribution specified by the subcommand. For example,

```
MTB > random 100 c1;
SUBC> bernoulli .5.
```

simulates the tossing of a fair coin 100 times and places the results in C1 using the **bernoulli** subcommand. If no subcommand is provided, this distribution is taken to be the $N(0, 1)$ distribution. The command

```
MTB > random 200 c1;
SUBC> normal mu=2.1 sigma=3.3.
```

generates a sample of 200 from the $N(2.1, 3.3)$ distribution using the **normal** subcommand. There are a number of other subcommands specifying distributions, and we refer the reader to **help** for a description of these.

3.3 Exercises

When the data for an exercise come from an exercise in IPS, the IPS exercise number is given in parentheses ( ). All computations in these exercises are to be carried out using Minitab, and the exercises are designed to ensure that you have a reasonable understanding of the Minitab material in this chapter. Generally, you should be using Minitab to do all the computations and plotting required for the problems in IPS.

If your version of Minitab places restrictions such that the value of the simulation sample size N requested in these problems is not feasible, then substitute a more appropriate value. Be aware, however, that the accuracy of your results is dependent on how large N is.

1. (3.13) Generate a random permutation of the names using Minitab.

2. (3.32) Use the Manip I Sort command described in I.11.6 to order the subjects by weight. Use the values 1–5 to indicate five blocks of equal length in a separate column, and then use the Manip I Unstack command described in I.11.7 to put the blocks in separate columns. Generate a random permutation of each block.

3. Use the following methodology to generate a sample of 20 from a population of 100,000. First, put the values 0–9 in each of C1–C5. Next, use sampling with replacement to generate 50 values from C1, and put the results in C6. Do the same for each of C2–C5 and put the results in C7–C10 (don't generate from these columns simultaneously). Create a single column of numbers using the digits in C6–C10 as the digits in the numbers. Pick out the first unique 20 entries as labels for the sample. If you do not obtain 20 unique values, repeat the process until you do. Why does this work?

4. Suppose you wanted to carry out stratified sampling where there are 3 strata, with the first stratum containing 500 elements, the second stratum containing 400 elements, and the third stratum containing 100 elements. Generate a stratified sample with 50 elements from the first stratum, 40 elements from the second stratum, and 10 elements from the third stratum. When the strata sample sizes are the same proportion of the total sample size as the strata population sizes are of the total population size this is called *proportional sampling*.

5. Suppose we have an urn containing 100 balls with 20 labeled 1, 50 labeled 2, and 30 labeled 3. Using sampling with replacement, generate a sample of size 1000 from this distribution employing the Calc I Random Data command to generate the sample directly from the relevant population distribution. Use the Stat I Tables I Cross Tabulation command to record the proportion of each label in the sample.

6. Carry out a simulation study with $N = 1000$ of the sampling distribution of $\hat{p}$ for $n = 5, 10, 20$ and for $p = .5, .75, .95$. In particular, calculate the empirical distribution functions and plot the histograms. Comment on your findings.

7. Carry out a simulation study with $N = 2000$ of the sampling distribution of the sample standard deviation when sampling from the $N(0,1)$ distribution based on a sample of size $n = 5$. In particular, plot the histogram using cutpoints 0, 1.5, 2.0 2.5, 3.0 5.0. Repeat this for the sample coefficient of variation (sample standard deviation divided by the sample mean) using the cutpoints -10, -9, ..., 0, ..., 9, 10. Comment on the shapes of the histograms relative to an $N(0,1)$ density curve.

Chapter 4

Probability: The Study of Randomness

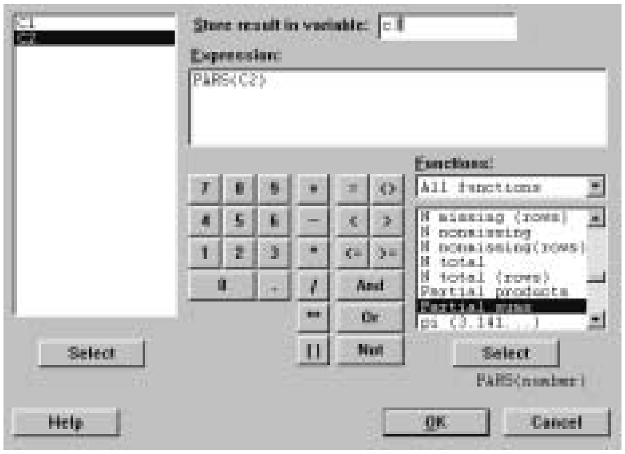
In this chapter the concept of probability is introduced more formally than previously in the book. Probability theory underlies the powerful computational methodology known as simulation, which we introduced in Chapter 3. Simulation has many applications in probability and statistics and also in many other fields, such as engineering, chemistry, physics, and economics.

4.1 Basic Probability Calculations

The calculation of probabilities for random variables can often be simplified by tabulating the cumulative distribution function. Also, means and variances are easily calculated using component-wise column operations in Minitab. For example, suppose we have the probability distribution

x	1	2	3	4
probability	.1	.2	.3	.4

in columns C1 and C2, with the values in C1 and the probabilities in C2. The Calc I Calculator command with the dialog box as in Display 4.1 computes the cumulative distribution function in C3 using Partial Sums.



Store result in variable: c3

Expression: PARS(C2)

Functions: All functions

Display 4.1: Dialog box for computing partial sums of entries in C2 and placing these sums in C3.

Printing C1 and C3 gives

```
Row   C1    C3
1      1   0.1
2      2   0.3
3      3   0.6
4      4   1.0
```

in the Session window. We can also easily compute the mean and variance of this distribution. For example, the session commands

```
MTB > let c4=c1*c2
MTB > let c5=c1*c1*c2
MTB > let k1=sum(c4)
MTB > let k2=sum(c5)-k1*k1
MTB > print k1 k2
K1 3.00000
K2 1.00000
```

calculate the mean and variance and store these in K1 and K2, respectively. The mean is 3 and the variance is 1. Of course, we can also use Calc I Calculator to do these calculations. In presenting more extensive computations, it is somewhat easier to list the appropriate session commands, as we will do subsequently. However, this is not to be interpreted as the required way to do these computations, as it is obvious that the menu commands can be used as well. Use whatever you find most convenient.

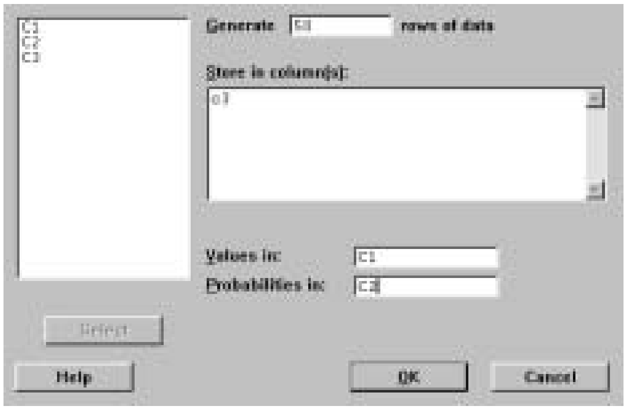
4.2 More on Sampling from Distributions

As we saw in II.3.2, Minitab includes algorithms for generating from many probability distributions using Calc I Random Data. This menu command produces a drop-down list that includes the normal, binomial, Chi-square, *F, t,* uniform, and many other distributions that the text, and this manual, will discuss. Clicking on one of these names results in a dialog box with entries to be filled in further specifying the distribution and the size of the sample.

For example, we can generate from one particularly important class of probability distributions using Calc I Random Data I Discrete. These probability distributions are concentrated on a finite number of values. To illustrate this, suppose we have the following values in C1 and C2.

```
Row   C1    C2
1     -1   0.3
2      2   0.2
3      3   0.4
4     10   0.1
```

Here, C1 contains the possible values of an outcome, and C2 contains the probabilities that each of these values is obtained, so, for example, $P(\{-1\}) = .3, P(\{2\}) = .2$, etc. The dialog box of Display 4.2 generates a sample of 50 from this discrete distribution and stores the sample in C3.



Generate [illegible] rows of data

Store in column(s): c3

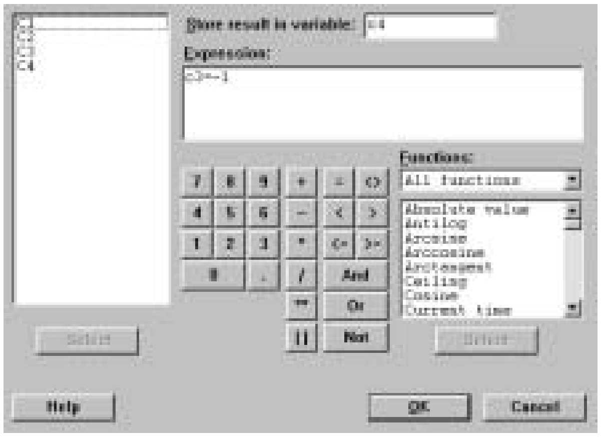
Values in: C1

Probabilities in: C2

Display 4.2: Dialog box for generating a sample from a discrete distribution with values in C1 and probabilities in C2 and storing the sample in C3.

It is an interesting exercise to check that the algorithms Minitab is using are in fact producing samples appropriately. There are a variety of things one could check, but perhaps the simplest is to check that the long-run relative frequencies are correct. So in the example of this section, we want to make sure that, as we increase the size of the sample, the relative frequencies of $-1, 2, 3, 10$ in the sample are getting closer to .3, .2, .4, and .1, respectively. Note that it is not guaranteed that as we increase the sample size that the relative frequencies get closer monotonically to the corresponding probabilities, but inevitably this must be the case.

First, we generated a sample of size 100 from this distribution and stored the values in C3 as in Display 4.2. Next, we recorded a 1 in C4 whenever the corresponding entry in C3 was -1 and recorded a 0 in C4 otherwise. To do this, we used the Calc I Calculator command with dialog box as shown in Display 4.3.



Store result in variable: C4

Expression: c3=-1

Functions: All functions

Display 4.3: Dialog box to record the incidence of a -1 in C3.

It is clear that the mean of C4 is the relative frequency of -1 in the sample. We calculated this mean using Calc I Column Statistics, as discussed in I.10.3, which gave the output

```
Mean of C4 = 0.33000
```

in the Session window. Repeating this with a sample of size 1000, we obtained

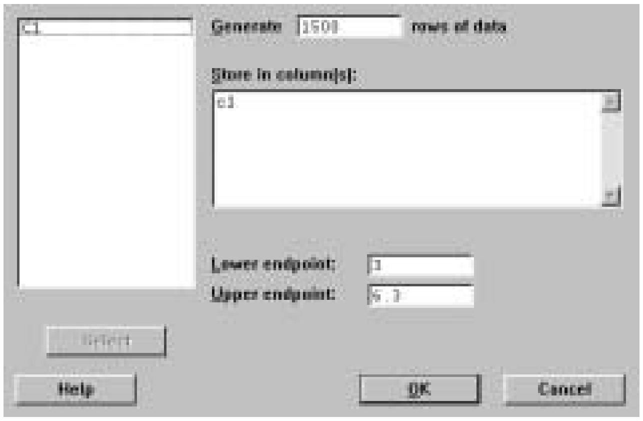
```
Mean of C4 = 0.28100
```

which we can see is a bit closer to the true value of .3. Repeating this with a sample of size $10,000$ from this distribution, we obtained

```
Mean of C4 = 0.29300
```

which is closer still. It would appear that the relative frequency of -1 is indeed converging to .3.

We can generate a randomly chosen point from the line interval (a, b) , where $a < b,$ using Calc I Random Data I Uniform. For example, the dialog box of Display 4.4 generates a sample of 1500 from the uniform distribution on the interval $(3.0, 6.3)$. With this distribution, the probability of any subinterval (c, d) of (a, b) is given by $(d - c) / (b - a),$ i.e., the length of (c, d) over the length of $(a, b).$ Of course, we can estimate this probability by just counting the number of times the generated response falls in the interval (c, d) and dividing this by the total sample size. For example, using the outcomes from the dialog box of Display 4.3 and estimating the probability of the interval $(4, 5)$, we get the relative frequency 0.30867, which is close to the true value of $(5 - 4) / (6.3 - 3) = 0.30303$.



C1

Generate 1500 rows of data

Store in column(s):
c1

Lower endpoint: 3
Upper endpoint: 6.3

Display 4.4: Dialog box for generating a sample of 1500 from the uniform distribution on the interval $(3.0, 6.3)$.

We can generalize this to generate from a point randomly chosen from a rectangle $(a, b) \times (c, d)$, i.e., the set of all points (x, y) such that $a < x < b, c < y < d$. If we want a sample of n from this distribution, we generate a sample $x_1, \ldots, x_n$ from the uniform on (a, b) and also generate a sample $y_1, \ldots, y_n$ from the uniform distribution on (c, d). Then $(x_1, y_1), \ldots, (x_n, y_n)$ is a sample of n from the uniform distribution on $(a, b) \times (c, d)$. We can approximate the probability of a random pair (x, y) falling in any subset $A \subset (a, b) \times (c, d)$ by computing the relative frequency of A in the sample.

The **random** command is the session command for carrying out simulations in Minitab. For example, the subcommand

uniform V_1 V_2

specifies the continuous uniform distribution on the interval $(\mathrm{V}_1, \mathrm{V}_2)$; i.e., subintervals of the same length have the same probability of occurring. If we have placed a discrete probability distribution in column E_2, on the values in column E_1, the subcommand

discrete E_1 E_2

generates a sample from this distribution.

4.3 Simulation for Approximating Probabilities

As previously noted, simulation can be used to approximate probabilities. For a variety of reasons, these simulations are most easily presented using session commands but it is clear that we can replace each step by the appropriate menu command.

For example, suppose we are asked to calculate

$$P(.1 \leq X_1 + X_2 \leq .3)$$

when X_1, X_2 are both independent and follow the uniform distribution on the interval $(0, 1)$. The session commands

```
MTB > random 1000 c1 c2;
SUBC> uniform 0 1.
MTB > let c3=c1+c2
MTB > let c4 = .1<=c3 and c3<=.3
MTB > let k1=sum(c4)/n(c4)
MTB > print k1
K1 0.0400000
MTB > let k2=sqrt(k1*(1-k1)/n(c4))
MTB > print k2
K2 0.00619677
MTB > let k3=k1-3*k2
MTB > let k4=k1+3*k2
MTB > print k3 k4
K3 0.0214097
K4 0.0585903
```

generate $N = 1000$ independent values of X_1, X_2 and place these values in C1 and C2, respectively, then calculate the sum $X_1 + X_2$ and put these values in C3. Using the comparison operators discussed in I.10.4, a 1 is recorded in C4 every time $.1 \le X_1 + X_2 \le .3$ is true and a 0 is recorded there otherwise. We then calculate the proportion of 1's in the sample as K1, and this is our estimate $\hat{p}$ of the probability. We will see later that a good measure of the accuracy of this estimate is the *standard error of the estimate*, which in this case is given by

$$\sqrt{\hat{p}\,(1 - \hat{p})\,/N}$$

and this is computed in K2. Actually, we can feel fairly confident that the true value of the probability is in the interval

$$\hat{p} \pm 3\sqrt{\hat{p}\,(1 - \hat{p})\,/N}$$

which in this case, equals the interval $(0.0214097, 0.0585903)$. So we know the true value of the probability with reasonable accuracy. As the simulation size N increases, the Law of Large Numbers says that $\hat{p}$ converges to the true value of the probability.

4.4 Simulation for Approximating Means

The means of distributions can also be approximated using simulations in Minitab. For example, suppose X_1, X_2 are both independent and follow the uniform distribution on the interval $(0, 1)$ and that we want to calculate the mean of $Y = 1/\,(1 + X_1 + X_2)$. We can approximate this in a simulation. The session commands

```
MTB > random 1000 c1 c2;
SUBC> uniform 0 1.
MTB > let c3=1/(1+c1+c2)
MTB > let k1=mean(c3)
MTB > let k2=stdev(c3)/sqrt(n(c3))
MTB > print k1 k2
K1 0.521532
K2 0.00375769
MTB > let k3=k1-3*k2
MTB > let k4=k1+3*k2
MTB > print k3 k4
K3 0.510259
K4 0.532805
```

generate $N = 1000$ independent values of X_1, X_2 and place these values in C1, C2, then calculate $Y = 1/(1 + X_1 + X_2)$ and put these values in C3. The mean of C3 is stored in K1, and this is our estimate of the mean value of Y. As a measure of how accurate this estimate is, we compute the standard error of the estimate, which is given by the standard deviation divided by the square root of the simulation sample size N. Again, we can feel fairly confident that the interval given by the estimate plus or minus 3 times the standard error of the estimate contains the true value of the mean. In this case, this interval is given by $(0.510259, 0.532805)$, and so we know this mean with reasonable accuracy. As the simulation size N increases, the Law of Large Numbers says that the approximation converges to the true value of the mean.

4.5 Exercises

When the data for an exercise come from an exercise in IPS, the IPS exercise number is given in parentheses ( ). All computations in these exercises are to be carried out using Minitab, and the exercises are designed to ensure that you have a reasonable understanding of the Minitab material in this chapter. Generally, you should be using Minitab to do all the computations and plotting required for the problems in IPS.

If your version of Minitab places restrictions such that the value of the simulation sample size N requested in these problems is not feasible, then substitute a more appropriate value. Be aware, however, that the accuracy of your results is dependent on how large N is.

1. Suppose we have the probability distribution

x	1	2	3	4	5
probability	.15	.05	.33	.37	.10

on the values 1, 2, 3, 4, and 5. Calculate the mean and variance of this distribution. Suppose that three independent outcomes (X_1, X_2, X_3) are generated from this distribution. Compute the probability that $1 < X_1 \leq 4, 2 \leq X_2$ and $3 < X_3 \leq 5$.

2. Suppose we have the probability distribution

x	1	2	3	4	5
probability	.15	.05	.33	.37	.10

on the values 1, 2, 3, 4, and 5. Using Minitab, verify that this is a probability distribution. Make a bar chart (probability histogram) of this distribution. Generate a sample of size 1000 from this distribution and plot a relative frequency histogram for the sample.

3. (4.23) Indicate how you would simulate the game of roulette using Minitab. Based on a simulation of $N = 1000$, estimate the probability of getting red and a multiple of 3.

4. A probability distribution is placed on the integers 1, 2, ..., 100, where the probability of integer i is c/i^2. Determine c so that this is a probability distribution. What is the 90th percentile? Generate a sample of 20 from the distribution.

5. Suppose an outcome is random on the square $(0, 1) \times (0, 1)$. Using simulation, approximate the probability that the first coordinate plus the second coordinate is less than .75 but greater than .25.

6. Gene©rate a sample of 1000 from the uniform distribution on the unit disk $D = (x, y) : x^2 + y^2 \leq 1$.

7. The expression e^{-x} for $x > 0$ is the density curve for what is called the *Exponential* (1) distribution. Plot this density curve in the interval from 0 to 10 using an increment of .1. The Calc I Random Data I Exponential command can be used to generate from this distribution by specifying the Mean as 1 in the ensuing dialog box. Generate a sample of 1000 from this distribution and estimate its mean. Approximate the probability that a value generated from this distribution is in the interval (1,2). The general *Exponential* (λ) has a density curve given by $\lambda^{-1}e^{-x/\lambda}$ for $x > 0$ and where $\lambda > 0$ is the mean. Repeat the simulation with mean $\lambda = 3$. Comment on the values of the estimated means.

8. Suppose you carry out a simulation to approximate the mean of a random variable X and you report the value 1.23 with a standard error of .025. If you are asked to approximate the mean of $Y = 3 + 5X$, do you have to carry out another simulation? If not, what is your approximation, and what is the standard error of this approximation?

9. Suppose that a random variable X follows an $N(3, 2.3)$ distribution. Subsequently, conditions change and no values smaller than -1 or bigger than 9.5 can occur, i.e., the distribution is conditioned to the interval $(-1, 9.5)$.

Generate a sample of 1000 from the truncated distribution, and use the sample to approximate its mean.

10. Suppose that X is a random variable and follows an $N(0,1)$ distribution. Simulate $N = 1000$ values from the distribution of $Y = X^2$, and plot these values in a histogram with cutpoints 0, .5, 1, 1.5, ..., 15. Approximate the mean of this distribution. Generate Y directly from its distribution, which is known to be a $Chisquare(1)$ distribution. In general, the $Chisquare(k)$ distribution can be generated from via the command Calc I Random Data I Chi-Square, where k is specified as the Degrees of freedom in the dialog box. Plot the Y values in a histogram using the same cutpoints. Comment on the two histograms. Note that you can plot the density curve of these distributions using Calc I Probability Distributions I Chi-Square and evaluating the probability density at a range of points as we discussed in II.2 for the normal distribution.

11. If X_1 and X_2 are independent random variables with X_1 following a $Chisquare(k_1)$ distribution and X_2 following a $Chisquare(k_2)$ distribution, then it is known that $Y = X_1 + X_2$ follows a $Chisquare(k_1 + k_2)$ distribution. For $k_1 = 1$, $k_2 = 1$, verify this empirically by plotting histograms with cutpoints 0, .5, 1, 1.5, ..., 15, based on simulations of size $N = 1000$.

12. If X_1 and X_2 are independent random variables with X_1 following an $N(0,1)$ distribution and X_2 following a $Chisquare(k)$ distribution, then it is known that
$$Y = \frac{X_1}{\sqrt{X_2/k}}$$
follows a $Student(k)$ distribution. The $Student(k)$ distribution can be generated from using the command Calc I Random Data I t, where k is the Degrees of freedom and must be specified in the dialog box. For $k = 3$, verify this result empirically by plotting histograms with cutpoints -10, -9, ..., 9, 10, based on simulations of size $N = 1000$.

13. If X_1 and X_2 are independent random variables with X_1 following a $Chisquare(k_1)$ distribution and X_2 following a $Chisquare(k_2)$ distribution, then it is known that
$$Y = \frac{X_1/k_1}{X_2/k_2}$$
follows an $F(k_1, k_2)$ distribution. The $F(k_1, k_2)$ distribution can be generated from using the subcommand Calc I Random Data I F, where k_1 is the Numerator degrees of freedom and k_2 is the Denominator degrees of freedom, both of which must be specified in the dialog box. For $k_1 = 1$, $k_2 = 1$, verify this empirically by plotting histograms with cutpoints 0, .5, 1, 1.5, ..., 15, based on simulations of size $N = 1000$.

Chapter 5

Sampling Distributions

New Minitab command discussed in this chapter

Calc I Probability Distributions I Binomial

Once data have been collected, they are analyzed using a variety of statistical techniques. Virtually, all of these involve computing *statistics* that measure some aspect of the data concerning questions we wish to answer. The answers determined by these statistics are subject to the uncertainty caused by the fact that we typically do not have the full population but only a sample from the population. As such, we have to be concerned with the variability in the answers when different samples are obtained. This leads to a concern with the *sampling distribution* of a statistic.

Sometimes, the sampling distribution of a statistic can be worked out exactly through various mathematical techniques, e.g., in Chapter 5 of IPS it is seen that the number of 1's in a sample of n from a $Bernoulli(p)$ distribution is $Binomial(n, p)$. Often, however, this is not possible, and we must resort to approximations. One approximation technique is to use simulation. Sometimes, however, the statistics we are concerned with are averages, and, in such cases, we can typically approximate their sampling distribution via an appropriate normal distribution.

5.1 The Binomial Distribution

Suppose that $X_1, \ldots, X_n$ is a sample from the $Bernoulli(p)$ distribution, i.e., $X_1, \ldots, X_n$ are independent realizations, where each X_i takes the value 1 or 0 with probabilities p and $1 - p$, respectively. The random variable $Y = X_1 + \cdots + X_n$ equals the number of 1's in the sample and follows, as discussed in IPS, a $Binomial(n, p)$ distribution. Therefore, Y can take on any of the values $0, 1, \ldots, n$ with positive probability. In fact, an exact formula can be derived for these probabilities; namely

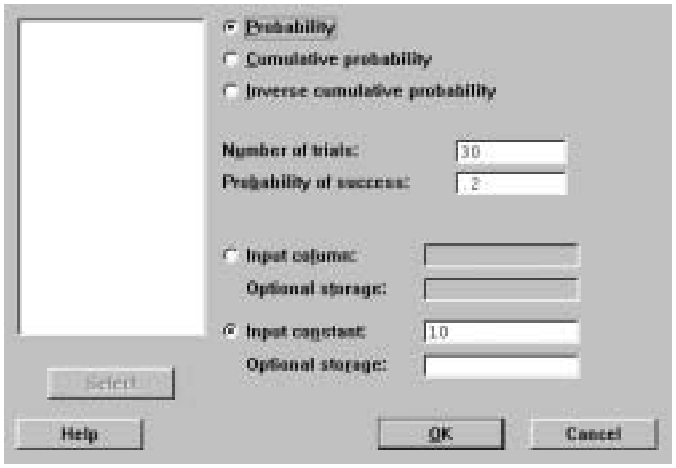
$$P(Y = k) = \binom{n}{k} p^k(1-p)^{n-k}$$

is the probability that Y takes the value k for $0 \le k \le n$. When n and k are small, this formula could be used to evaluate this probability but it is almost always better to use software like Minitab to do it, and when these values are not small, it is necessary. Also, we can use Minitab to compute the $Binomial(n, p)$ cumulative probability distribution — the probability contents of intervals $(-\infty, x]$ and the inverse cumulative distribution — percentiles of the distribution.

For individual probabilities, we use the Calc I Probability Distributions I Binomial command. For example, suppose we have a $Binomial(30, .2)$ distribution and want to compute the probability $P(Y = 10)$. This command, with the dialog box as in Display 5.1, produces the output

```
Binomial with n = 30 and p = 0.200000
   x       P( X = x )
 10.00      0.0355
```

in the Session window, i.e., $P(Y = 10) = .0355$.



● Probability
○ Cumulative probability
○ Inverse cumulative probability

Number of trials: 30
Probability of success: .2

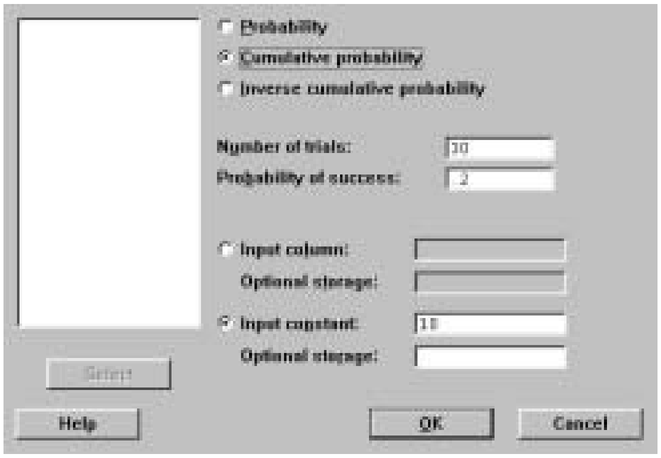
○ Input column:
Optional storage:
● Input constant: 10
Optional storage:

Display 5.1: Dialog box for $Binomial(n, p)$ probability calculations.

If we want to compute the probability of getting 10 or fewer successes, this is the probability of the interval $(-\infty, 10]$, and we can use the Calc I Probability Distributions I Binomial command with the dialog box as in Display 5.2. This produces the output

```
Binomial with n = 30 and p = 0.200000
  x        P( X <= x )
 10.00      0.9744
```

in the Session window, i.e., $P(Y \le 10) = .9744$.



○ Probability
● Cumulative probability
○ Inverse cumulative probability

Number of trials: 30
Probability of success: .2

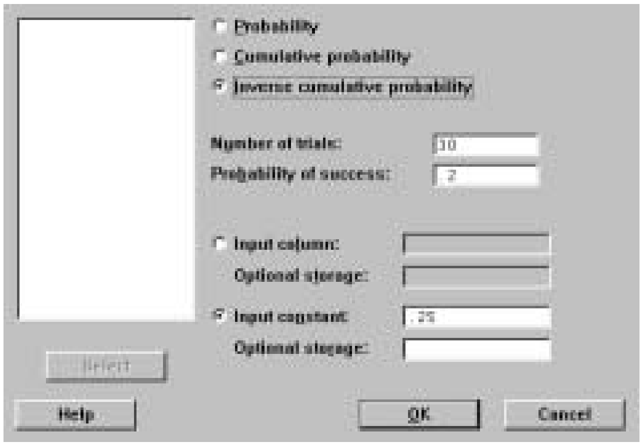
○ Input column:
Optional storage:
● Input constant: 10
Optional storage:

Display 5.2: Dialog box for computing cumulative probabilities for the $Binomial(n, p)$ distribution.

Suppose we want to compute the first quartile of this distribution. The Calc I Probability Distributions I Binomial command, with the dialog box as in Display 5.3, produces the output

```
Binomial with n = 30 and p = 0.200000
 x    P( X <= x )    x      P( X <= x )
 3      0.1227       4        0.2552
```

in the Session window. This gives the values x that have cumulative probabilities just smaller and just larger than the value requested. Recall that with a discrete distribution, such as the $Binomial(n, p)$, we will not in general be able to obtain an exact percentile.



○ Probability
○ Cumulative probability
● Inverse cumulative probability

Number of trials: 30
Probability of success: .2

○ Input column:
Optional storage:
● Input constant: .25
Optional storage:

Display 5.3: Dialog box for computing percentiles of the $Binomial(n, p)$ distribution.

These commands can operate on all the values in a column simultaneously. This is very convenient if you should want to tabulate or graph the probability function, cumulative distribution function, or inverse distribution function.

The general syntax of the **pdf, cdf,** and **invcdf** session commands is given in II.1.3, and here we use them with the **binomial** subcommand as in

```
MTB > pdf 10;
SUBC> binomial 30 .2.
```

which outputs $P(Y = 10)$ when Y has the $Binomial(30, .2)$ distribution.

Actually, when n is very large even software will not be useful to compute these probabilities, and you will have to use normal approximations to binomial probabilities via the central limit theorem. The **pdf** and **cdf** commands with the **normal** subcommand can be used for this.

We might also want to simulate from the $Binomial(n, p)$ distribution. For this we use the Calc I Random Data I Binomial command or the session command **random** with the **binomial** subcommand. For example,

```
MTB > random 10 c1;
SUBC> binomial 30 .2.
MTB > print c1
C1
 2 2 4 2 11 5 7 8 5 2
```

generates a sample of 10 from the $Binomial(30, .2)$ distribution.

5.2 Simulating Sampling Distributions

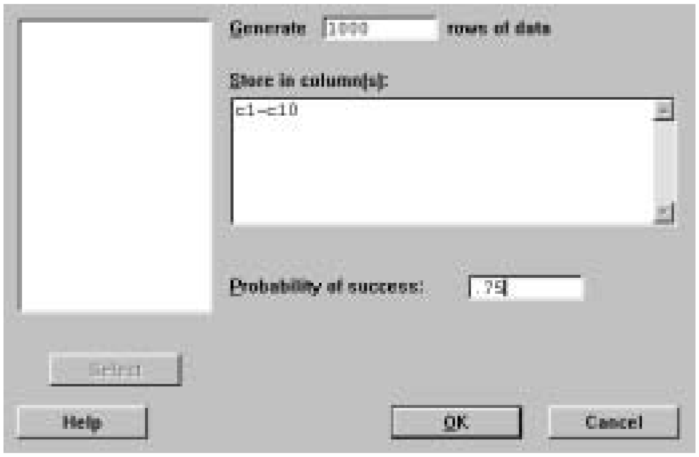
First, we consider an example where we know the exact sampling distribution. Suppose we flip a possibly biased coin n times and want to estimate the unknown probability p of getting a head. The natural estimate is $\hat{p}$ the proportion of heads in the sample. We would like to assess the sampling behavior of this statistic in a simulation. To do this, we choose a value for $p,$ then generate N samples from the Bernoulli distribution of size $n,$ for each of these compute $\hat{p},$ look at the empirical distribution of these N values, perhaps plotting a histogram as well. The larger N is the closer the empirical distribution and histogram will be to the true sampling distribution of $\hat{p}$.

Note that there are two sample sizes here: the sample size n of the original sample the statistic is based on, which is fixed, and the *simulation* sample size $N,$ which we can control. This is characteristic of all simulations. Sometimes, using more advanced analytical techniques we can determine N so that the sampling distribution of the statistic is estimated with some prescribed accuracy. Some techniques for doing this are discussed in later chapters of IPS. Another method is to repeat the simulation a number of times, slowly increasing N until we see the results stabilize. This is sometimes the only way available, but caution should be shown as it is easy for simulation results to be very misleading if the final N is too small.

We illustrate a simulation to determine the sampling distribution of $\hat{p}$ when sampling from a *Bernoulli*(.75) distribution. For this, we use the commands Calc I Random Data I Bernoulli, Calc I Row Statistics, and Stat I Tables I Tally, with the dialog boxes given by Displays 5.4, 5.5, and 5.6, respectively, to produce the output

```
Summary Statistics for Discrete Variables
 C11 CumPct
 0.3   0.40
 0.4   2.20
 0.5   7.60
 0.6  23.10
 0.7  47.70
 0.8  78.00
 0.9  94.70
 1.0 100.00
```

in the Session window. Here we have generated $N = 1000$ samples of size $n = 10$ from the *Bernoulli*(.75) distribution, i.e., we simulated the tossing of this coin 10,000 times, and we placed the results in the rows of columns C1–C10 using Calc I Random Data I Bernoulli. The proportion of heads $\hat{p}$ in each sample is computed and placed in C11 using Calc I Row Statistics. Note that a mean of values equal to 0 or 1 is just the proportion of 1's in the sample. Finally, we used Stat I Tables I Tally to compute the empirical distribution function of these 1000 values of $\hat{p}$. For example, this says 78% of these values were .8 or smaller and there were no instances smaller than .3.

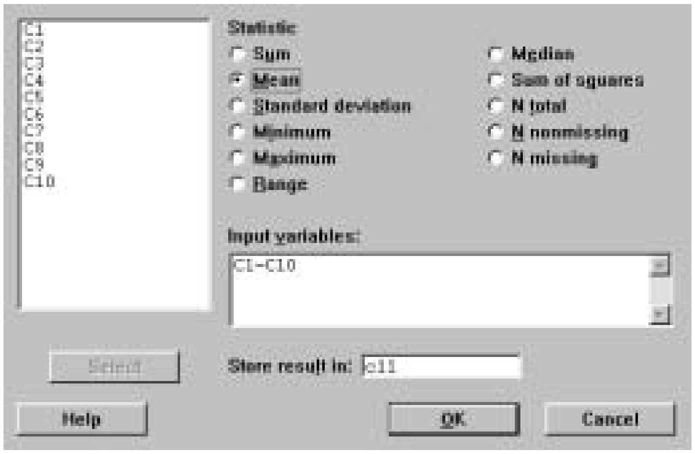


Generate 1000 rows of data

Store in column(s):
c1-c10

Probability of success: .75

Display 5.4: Dialog box for generating 10 columns of 1000 *Bernoulli*(.75) values.

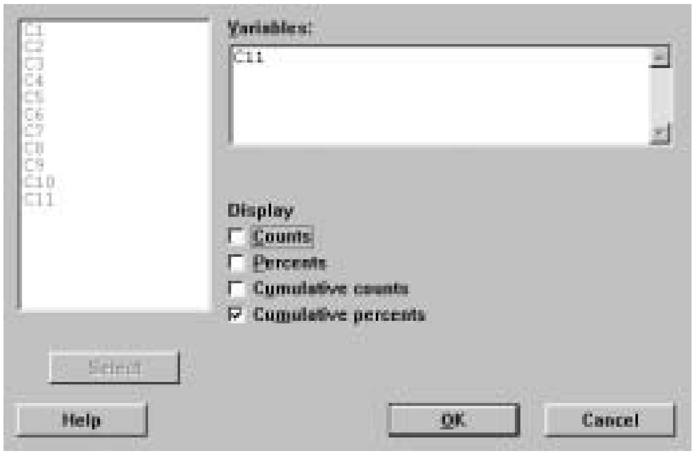


Statistic
○ Sum
● Mean
○ Standard deviation
○ Minimum
○ Maximum
○ Range
○ Median
○ Sum of squares
○ N total
○ N nonmissing
○ N missing

Input variables: C1-C10

Store result in: c11

Display 5.5: Dialog box for computing the proportion of 1's in each of the 1000 samples of size 10.



Variables: C11

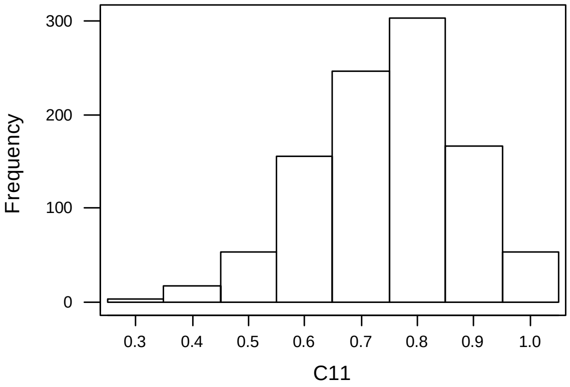
Display
☐ Counts
☐ Percents
☐ Cumulative counts
☑ Cumulative percents

Display 5.6: Dialog box for computing the empirical distribution function of $\hat{p}$.

In Display 5.7, we have plotted a histogram of the 1000 values of $\hat{p}$. Based on N = 800, the following empirical distribution was obtained:

```
C11  CumPct
 0.4   1.20
 0.5   7.20
 0.6  22.20
 0.7  47.80
 0.8  78.20
 0.9  95.00
 1.0 100.00
```

Because these values are reasonably close to those obtained with N = 1000, we stopped at N = 1000.



Display 5.7: Histogram of simulation of $N = 1000$ values of $\hat{p}$ based on a sample of size $n = 10$ from the Bernoulli(.75) distribution.

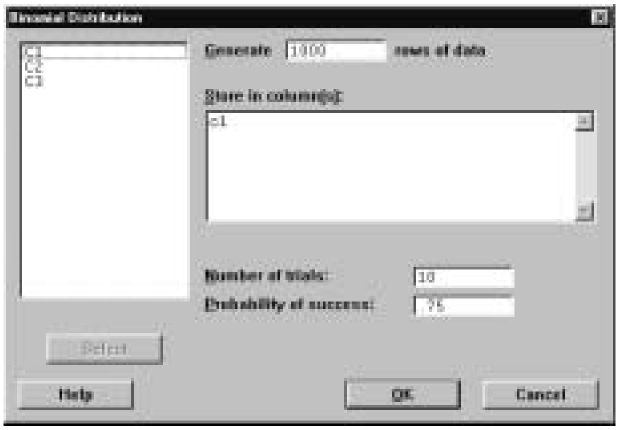
The corresponding session commands for this simulation are

```
MTB > random 1000 c1-c10;
SUBC> bernoulli .75.
MTB > rmean c1-c10 c11
MTB > tally c11;
SUBC> cumpcts.
```

and these might seem like an easier way to implement the simulation.

In Chapter 5 of IPS we saw that the sampling distribution of $\hat{p}$ can be determined exactly, i.e., there are formulas to determine this, and we can simulate directly from the sampling distribution, so this simulation can be made much more efficient. In effect, this entails using the Calc I Random Data I Binomial command with dialog box as in Display 5.8 and dividing each entry in C1 by 10. This generates $N = 1000$ values of $\hat{p}$ but uses a much smaller number of cells. Still, there are many statistics for which this kind of efficiency reduction is not available, and, to get some idea of what their sampling distribution is like, we must resort to the more brute force form of simulation of generating directly from the population distribution.

Sometimes, more sophisticated simulation techniques are needed to get an accurate assessment of a sampling distribution. Within Minitab, there are programming techniques, which we do not discuss in this manual, that can be applied in such cases. For example, it is clear that if our simulation required the generation of 10^6 cells (and this is not at all uncommon for some harder problems), the simulation approach we have described would not work within Minitab, as the worksheet would be too large.



Generate 1000 rows of data

Store in column(s): c1

Number of trials: 10

Probability of success: .75

Display 5.8: Dialog box for generating 1000 values from the sampling distribution of $10\hat{p}$ using the $Binomial(10, .75)$ distribution.

5.3 Exercises

When the data for an exercise come from an exercise in IPS, the IPS exercise number is given in parentheses ( ). All computations in these exercises are to be carried out using Minitab, and the exercises are designed to ensure that you have a reasonable understanding of the Minitab material in this chapter. Generally, you should be using Minitab to do all the computations and plotting required for the problems in IPS.

If your version of Minitab places restrictions such that the value of the simulation sample size N requested in these problems is not feasible, then substitute a more appropriate value. Be aware, however, that the accuracy of your results is dependent on how large N is.

1. Calculate all the probabilities for the $Binomial(5, .4)$ distribution and the $Binomial(5, .6)$ distribution. What relationship do you observe? Can you explain this and state a general rule?

2. Compute all the probabilities for a $Binomial(5, .8)$ distribution and use these to directly calculate the mean and variance. Verify your answers using the formulas provided in IPS.

3. Compute and plot the probability and cumulative distribution functions of the $Binomial\,(10, .2)$ and the $Binomial\,(10, .5)$ distributions. Comment on the shapes of these distributions.

4. Generate 1000 samples of size 10 from the $Bernoulli(.3)$ distribution. Compute the proportion of 1's in each sample and compute the proportion of samples having no 1's, one 1, two 1's, etc. Compute what these proportions would be in the longrun and compare.

5. Carry out a simulation study with $N = 1000$ of the sampling distribution of $\hat{p}$ for $n = 5, 10, 20$ and for $p = .5, .75, .95$. In particular, calculate the empirical distribution functions and plot the histograms. Comment on your findings.

6. Suppose that $X_1, X_2, X_3, \ldots$ are independent realizations from the *Bernoulli*(p) distribution, i.e., each X_i takes the value 1 or 0 with probabilities p and $1 - p,$ respectively. If the random variable Y counts the number of tosses until we obtain the first head in a sequence of independent tosses $X_1, X_2, X_3, \ldots,$ then Y has a *Geometric*(p) distribution. Minitab does not have built-in algorithms for computing the probability function, distribution function, inverse distribution function, and for generating from this distribution. The probability function for this distribution is given by
$$P(Y = y) = (1 - p)^{y-1} p$$
for $y = 1, 2, \ldots$. Plot the probability function for the *Geometric* $(.5)$ distribution for the values $y = 1, \ldots, 10$. Do the same for the *Geometric* $(.1)$ distribution. What do you notice?

7. Using methods for summing geometric sums, the cumulative distribution function of the *Geometric* (p) distribution (see Exercise II.5.6) is given by $P(Y \leq y) = 1 - (1 - p)^y.$ Plot the cumulative distribution function for the *Geometric* $(.5)$ and *Geometric* $(.1)$ distribution for the values $y = 1, \ldots, 10$. What do you notice?

8. To randomly generate from the *Geometric*(p) distribution (see Exercise II.5.6), we can repeatedly generate from a *Bernoulli*(p) and count how many times we did this until the first 1 appeared. A simple way to do this in Minitab is to generate N values from the *Bernoulli*(p) into a column. Count the number of entries until the first 1, count the number of subsequent entries until the next 1, etc. These counts are identically and independently distributed according to the *Geometric*(p) distribution. This is a very inefficient method when p is small and much better algorithms exist. Generate a sample of 10 from the *Geometric* $(.5)$ distribution.

9. Carry out a simulation study, with $N = 2000$, of the sampling distribution of the sample standard deviation when sampling from the $N(0, 1)$ distribution, based on a sample of size $n = 5$. In particular, plot the histogram using cutpoints 0, 1.5, 2.0 2.5, 3.0 5.0. Repeat this for the sample coefficient of variation (sample standard deviation divided by the sample mean) using the cutpoints -10, -9, ..., 0, ..., 9, 10. Comment on the shapes of the histograms relative to a $N(0, 1)$ density curve.

10. Generate $N = 1000$ samples of size $n = 5$ from the $N(0, 1)$ distribution. Record a histogram for $\bar{x}$ using the cutpoints $-3, -2.5, -2, ..., 2.5, 3.0$. Generate a sample of size $N = 1000$ from the $N(0, 1/\sqrt{5})$ distribution. Plot the histogram using the same cutpoints and compare the histograms. What will happen to these histograms as we increase N?

11. Generate $N = 1000$ values of X_1, X_2, where X_1 follows a $N(3, 2)$ distribution and X_2 follows a $N(-1, 3)$ distribution. Compute $Y = X_1 - 2X_2$ for each of these pairs and plot a histogram for Y using the cutpoints $-20, -15, ..., 25, 30$. Generate a sample of $N = 1000$ from the appropriate distribution of Y and plot a histogram using the same cutpoints.

12. Plot the density curve for the *Exponential*(3) distribution (see Exercise II.4.7) between 0 and 15 with an increment of .1. Generate $N = 1000$ samples of size $n = 2$ from the *Exponential*(3) distribution and record the sample means. Standardize the sample of $\overline{x}$ using $\mu = 3$ and $\sigma = 3$. Plot a histogram of the standardized values using the cutpoints -5, -4, ..., 4, 5. Repeat this for $n = 5, 10$. Comment on the shapes of these histograms.

13. Plot the density of the uniform distribution on (0,1). Generate $N = 1000$ samples of size $n = 2$ from this distribution. Standardize the sample of $\overline{x}$ using $\mu = .5$ and $\sigma = \sqrt{1/12}$. Plot a histogram of the standardized values using the cutpoints $-5, -4, ..., 4, 5$. Repeat this for $n = 5, 10$. Comment on the shapes of these histograms.

14. The *Weibull*(β) has density curve given by $\beta x^{\beta-1} e^{-x^{\beta}}$ for $x > 0$, where $\beta > 0$ is a fixed constant. Plot the *Weibull*(2) density in the range 0 to 10 with an increment of .1 using the Calc I Probability Distributions I Weibull, command. Generate a sample of $N = 1000$ from this distribution using the subcommand Calc I Random Data I Weibull where β is the Shape parameter and the Scale parameter is 1. Plot a probability histogram and compare with the density curve.

Chapter 6

Introduction to Inference

New Minitab commands discussed in this chapter

Stat I Basic Statistics I 1-Sample Z
Power and Sample Size I 1-Sample Z

In this chapter, the basic tools of statistical inference are discussed. There are a number of Minitab commands that aid in the computation of confidence intervals and for carrying out tests of significance.

6.1 z-Confidence Intervals

The command Stat I Basic Statistics I 1-Sample Z computes confidence intervals for the mean μ using a sample $x_1, \ldots, x_n$ from a distribution where we know the standard deviation σ. There are three situations when this is appropriate:

(1) We know that we are sampling from a normal distribution with unknown mean μ and known standard deviation σ, and thus

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

is distributed $N(0, 1)$.
(2) We have a large sample from a distribution with unknown mean μ and known standard deviation σ, and the central limit theorem approximation to the distribution of $\bar{x}$ is appropriate, i.e., the distribution of

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

is approximately distributed $N(0, 1)$.

(3) We have a large sample from a distribution with unknown mean μ and unknown standard deviation σ, and the sample size is large enough so that

$$z = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

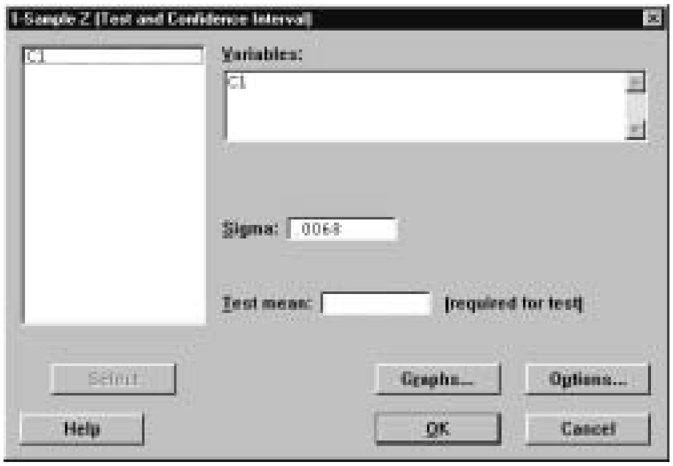
is approximately $N(0,1)$, where s is the sample standard deviation.

The confidence interval takes the form $\overline{x} \pm z^*\sigma/\sqrt{n}$, where s is substituted for σ in case (3), and z^* is determined from the $N(0,1)$ distribution by the confidence level desired, as described in IPS. Of course, situation (3) is probably the most realistic, but note that the confidence intervals constructed for (1) are exact, while those constructed under (2) and (3) are only approximate, and a larger sample size is required in (3) for the approximation to be reasonable than for (2).

Consider the sample given by 0.8403, 0.8363, 0.8447, which are stored in C1, and suppose that it makes sense to take $\sigma = .0068$. The command Stat I Basic Statistics I 1-Sample Z with the dialog boxes as in Displays 6.1 and 6.2 produces the output

```
Variable   N    Mean     StDev     SE Mean
  C1       3  0.84043   0.00420    0.00393
    99.0% CI
(0.83032, 0.85055)
```

in the Session window. This specifies the 99% confidence interval (0.83032, 0.85055) for μ. Note that in the dialog box of Display 6.1, we specify where the data resides in the Variables box, the value of σ in the Sigma box, and click on the Options button to bring up the dialog box in Display 6.2. In this dialog box we have specified the 99% confidence level in the Confidence level box.



1-Sample Z (Test and Confidence Interval)

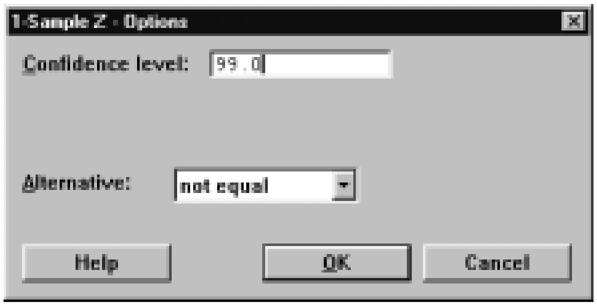
C1

Variables: C1

Sigma: .0068

Test mean: [required for test]

Display 6.1: First dialog box for producing the z-confidence interval for μ.



Confidence level: 99.0

Alternative: not equal

Display 6.2: Second dialog box for producing the z-confidence interval. Here we specify the confidence level.

The general syntax of the corresponding session command **zinterval** is

zinterval V_1 sigma $= \mathrm{V}_2$ $\mathrm{E}_1 \ldots \mathrm{E}_m$

where V_1 is the confidence level and is any value between 1 and 99.99, V_2 is the assumed value of σ, and E_1, ..., E_m are columns of data. A V_1% confidence interval is produced for each column specified. If no value is specified for V_1, the default value is 95%.

6.2 z-Tests

The Stat I Basic Statistics I 1-Sample Z command is used when we want to test the hypothesis that the unknown mean μ equals a value μ_0 and one of the situations (1), (2), or (3) as discussed in II.10.1 is appropriate. The test is based on computing a P-value using the observed value of

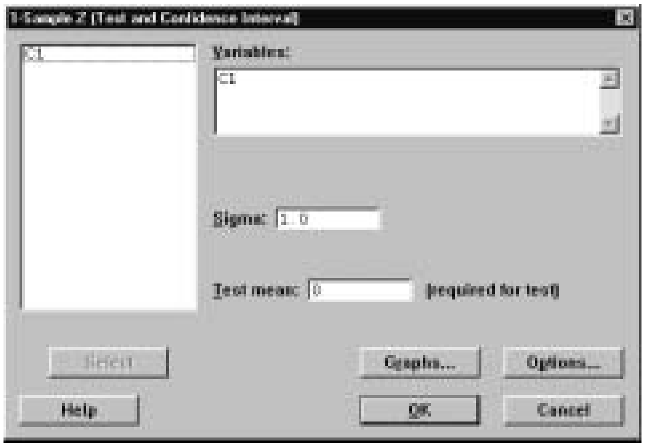
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

and the $N(0, 1)$ distribution as described in IPS.

Suppose the sample $2.0, 0.4, 0.7, 2.0, -0.4, 2.2, -1.3, 1.2, 1.1, 2.3$ is stored in C1, and we are asked to test the null hypothesis $H_0 : \mu = 0$ against the alternative $H_a : \mu > 0$ and it makes sense to take $\sigma = 1$. The Stat I Basic Statistics I 1-Sample Z command together with the dialog boxes of Displays 6.3 and 6.4 produces the output

```
Variable     99.0% Lower Bound        Z         P
   C1              0.284             3.23      0.001
```

in the Session window. This specifies the P-value for this test as .001, and so we reject the null hypothesis in favor of the alternative. In the first dialog box, we specified where the data is located, the value of σ as before and that we want to test $H_0 : \mu = 0$ by 0 in the Test mean box. We brought up the second dialog box by clicking on the Options button. In the second dialog box, we specified that we want to test this null hypothesis against the alternative $H_a : \mu > 0$ by selecting "greater than" in Alternative box. The other choices are "not equal," which selects the alternative $H_a : \mu \neq 0$, and "less than," which selects the alternative $H_a : \mu < 0$.



1-Sample Z (Test and Confidence Interval)

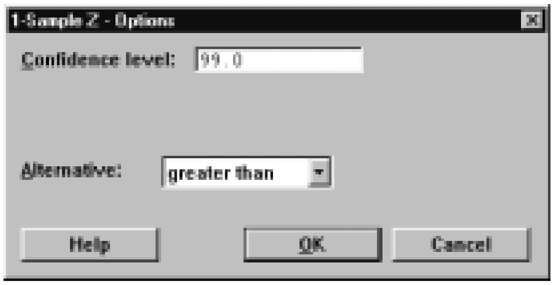
C1

Variables: C1

Sigma: 1.0

Test mean: 0 (required for test)

Display 6.3: First dialog box for testing a hypothesis concerning the mean using a z-test.



1-Sample Z - Options

Confidence level: 99.0

Alternative: greater than

Display 6.4: Second dialog box for testing a hypothesis using the z-test.

The general syntax of the corresponding session command **ztest** is

ztest V_1 sigma = V_2 $\mathrm{E}_1 \ldots \mathrm{E}_m$

where V_1 is the hypothesized value to be tested, V_2 is the assumed value of σ, and E_1, ..., E_m are columns of data. If no value is specified for V_1, the default is 0. A test of the hypothesis is carried out for each column. If no **alternative** subcommand is specified, a two-sided test is conducted, i.e., $H_0 : \mu = \mathrm{V}_1$ against the alternative $H_a : \mu \neq \mathrm{V}_1$. If the subcommand

```
SUBC> alternative 1.
```

is used, a test of $H_0 : \mu = \mathrm{V}_1$ against the alternative $H_a : \mu > \mathrm{V}_1$ is conducted. If the subcommand

```
SUBC> alternative -1.
```

is used, a test of $H_0 : \mu = \mathrm{V}_1$ against the alternative $H_a : \mu < \mathrm{V}_1$ is conducted.

6.3 Simulations for Confidence Intervals

When we are sampling from a $N(\mu, \sigma)$ distribution and know the value of $\sigma,$ the confidence intervals constructed in II.6.1 are exact, i.e., in the long run a proportion 95% of the 95% confidence intervals constructed for an unknown mean μ will contain the true value of this quantity. Of course, any given confidence interval may or may not contain the true value of $\mu,$ and, in any finite number of such intervals so constructed, some proportion other than 95% will contain the true value of μ. As the number of intervals increases, however, the proportion covering will go to 95%.

We illustrate this via a simulation study based on computing 90% confidence intervals. The session commands

```
MTB > random 100 c1-c5;
SUBC> normal 1 2.
MTB > rmean c1-c5 c6
MTB > invcdf .95;
SUBC> normal 0 1.
Normal with mean = 0 and standard deviation = 1.00000
 P( X <= x) x
 0.9500 1.6449
MTB > let k1=1.6449*2/sqrt(5)
MTB > let c7=c6-k1
MTB > let c8=c6+k1
MTB > let c9=c7<1 and c8>1
MTB > mean c9
 Mean of C9 = 0.94000
MTB > set c10
DATA> 1:25
DATA> end
MTB > delete 26:100 c7 c8
MTB > mplot c7 versus c10 c8 versus c10;
SUBC> xstart=1 end=25;
SUBC> xincrement=1.
```

generate 100 random samples of size 5 from the $N(1, 2)$ distribution, place the means in C6, the lower end-point of a 90% confidence interval in C7, and the upper end-point in C8, and record whether or not a confidence interval covers the true value $\mu = 1$ by placing a 1 or 0 in C9, respectively. The mean of C9 is the proportion of intervals that cover, and this is 94%, which is 4% too high. Finally, we plotted the first 25 of these intervals in a plot shown in Figure 6.1. Drawing a solid horizontal line at 1 on the y-axis indicates that most of these intervals do indeed cover the true value $\mu = 1$.

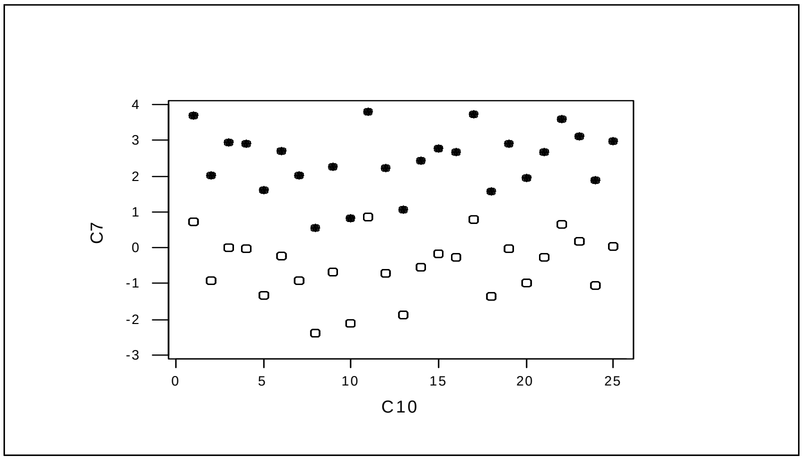


Figure 6.1: Plot of 90% confidence intervals for the mean when sampling from the $N(1,2)$ distribution with $n = 5$. The lower end-point is open and the upper end-point is closed.

The simulation just carried out simply verifies a theoretical fact. On the other hand, when we are computing approximate confidence intervals — i.e., we are not sampling necessarily from a normal distribution — it is good to do some simulations from various distributions to see how much reliance we can place in the approximation at a given sample size. The true *coverage probability* of the interval, i.e., the long-run proportion of times that the interval covers the true mean, will not in general be equal to the nominal confidence level. Small deviations are not serious, but large ones are.

6.4 Simulations for Power Calculations

It is also useful to know in a given context how sensitive a particular test of significance is. By this we mean how likely it is that the test will lead us to reject the null hypothesis when the null hypothesis is false. This is measured by the concept of the *power* of a test. Typically, a level α is chosen for the P-value at which we would definitely reject the null hypothesis if the P-value is smaller than α. For example, $\alpha = .05$ is a common choice for this level. Suppose that we have chosen the level of .05 for the two-sided z-test and we want to evaluate the power of the test when the true value of the mean is $\mu = \mu_1$, i.e., evaluate the probability of getting a P-value smaller than .05 when the mean is μ_1. The two-sided z-test with level α rejects $H_0 : \mu = \mu_0$ whenever

$$P\left(|Z| > \left|\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}\right|\right) \le \alpha$$

where Z is a $N(0,1)$ random variable. This is equivalent to saying that the null hypothesis is rejected whenever

$$\left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|$$

is greater than or equal to the $1-\alpha/2$ percentile for the $N(0,1)$ distribution. For example, if $\alpha = .05$, then $1-\alpha/2 = .975$ and this percentile can be obtained using the command Calc I Probability Distributions I Normal and the inverse distribution function, which gives the output

```
Normal with mean = 0 and standard deviation = 1.00000
 P( X <= x)        x
    0.9750      1.9600
```

in the Session window, i.e., the .975 percentile of the $N(0,1)$ distribution is 1.96. Denote this percentile by z^*. If $\mu = \mu_1$, then

$$\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$$

is a realized value from the distribution of $Y = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$ when $\bar{X}$ is distributed $N(\mu_1, \sigma/\sqrt{n})$. Therefore, Y follows a $N(\frac{\mu_1-\mu_0}{\sigma/\sqrt{n}}, 1)$ distribution. The power of the two-sided test at $\mu = \mu_1$ is

$$P(|Y| > z^*)$$

and this can be evaluated exactly using the command Calc I Probability Distributions I Normal and the distribution function, after writing

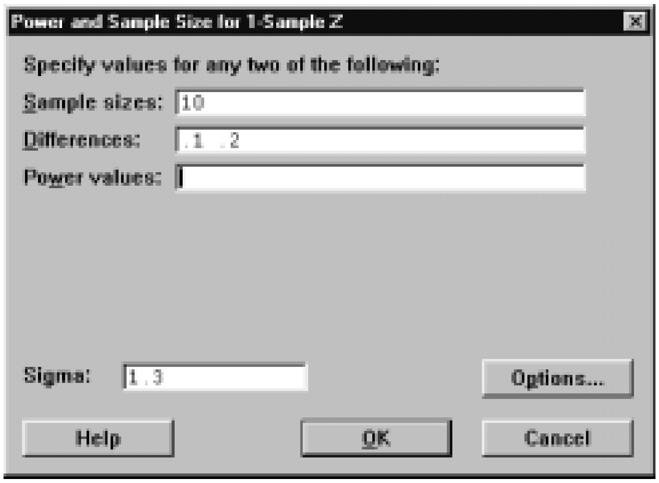
$$\begin{aligned} P(|Y| > z^*) &= P(Y > z^*) + P(Y < -z^*) \\ &= P\left(Z > -\frac{(\mu_1-\mu_0)}{\sigma/\sqrt{n}} + z^*\right) + P\left(Z < -\frac{(\mu_1-\mu_0)}{\sigma/\sqrt{n}} - z^*\right) \end{aligned}$$

with Z following an $N(0,1)$ distribution.

Alternatively, exact power calculations can be carried out under the assumption of sampling from a normal distribution using the Power and Sample Size I 1-Sample Z command and filling in the dialog box appropriately. Also, the minimum sample size required to guarantee a given power at a prescribed difference $|\mu_1 - \mu_0|$ can be obtained using this command. For example, filling in the dialog box for this command as in Display 6.5 creates the output

```
Testing mean = null (versus not = null)
Calculating power for mean = null + difference
Alpha = 0.05 Sigma = 1.3
 Sample
Difference Size   Power
 0.1        10   0.0568
 0.2        10   0.0775
```

in the Session window. This gives the power for testing $H_0 : \mu = \mu_0$ versus $H_0 : \mu \neq \mu_0$ at $|\mu_1 - \mu_0| = .1$ and $|\mu_1 - \mu_0| = .2$ when $n = 10$, $\sigma = 1.3$, and $\alpha = .05$. These powers are given by .0568 and .0775, respectively. Clicking on the Options button allows you to choose other alternatives and specify other values of α in the Significance level box.



Specify values for any two of the following:

Sample sizes: 10

Differences: .1 .2

Power values:

Sigma: 1.3

Display 6.5: Dialog box for calculating powers and minimum sample sizes.

If we had instead filled in Power values at .1 and .2 in the dialog box of Display 6.5, say as .8 and .9, and had left the Sample sizes box empty, we would have obtained the output

```
Testing mean = null (versus not = null)
Calculating power for mean = null + difference
Alpha = 0.05 Sigma = 1.3
 Sample Target Actual
  Difference     Size      Power     Power
         0.1     1327     0.8000    0.8002
         0.1     1776     0.9000    0.9000
         0.2      332     0.8000    0.8005
         0.2      444     0.9000    0.9000
```

in the Session window. This prescribes the minimum sample sizes $n = 1327$ and $n = 1776$ to obtain the powers .8 and .9, respectively, at the difference .1 and the sample sizes $n = 332$ and $n = 444$ to obtain the powers .8 and .9, respectively, at the difference .2.

This derivation of the power of the two-sided test depended on the sample coming from a normal distribution, as this leads to $\bar{X}$ having an exact normal distribution. In general, however, $\bar{X}$ will be only approximately normal, and so the normal calculation is not exact. To assess the effect of the nonnormality, however, we can often simulate sampling from a variety of distributions and estimate the probability $P(|Y| > z^*)$. For example, suppose that we want to test $H_0 : \mu = 0$ in a two-sided z-test based on a sample of 10, where we estimate σ by the sample standard deviation and we want to evaluate the power at 1. Let us further suppose that we are actually sampling from a uniform distribution on the interval $(-10, 12)$, which indeed has its mean at 1. The simulation given by the session commands

```
MTB > random 1000 c1-c10;
SUBC> uniform -10 12.
MTB > rmean c1-c10 c11
MTB > rstdev c1-c10 c12
MTB > let c13=absolute(c11/(c12/sqrt(10)))
MTB > let c14=c13>1.96
MTB > let k1=mean(c14)
MTB > let k2=sqrt(k1*(1-k1)/n(c14))
MTB > print k1 k2
K1 0.112000
K2 0.00997276
```

estimates the power to be .112, and the standard error of this estimate, as given in K2, is approximately .01. The application determines whether or not the assumption of a uniform distribution makes sense and whether or not this power is indicative of a sensitive test or not.

6.5 The Chi-Square Distribution

If Z is distributed according to the $N(0, 1)$ distribution, then $Y = Z^2$ is distributed according to the $Chisquare(1)$ distribution. If X_1 is distributed $Chisquare(k_1)$ independent of X_2 distributed $Chisquare(k_2)$, then $Y = X_1 + X_2$ is distributed according to the $Chisquare(k_1 + k_2)$ distribution. There are Minitab commands that assist in carrying out computations for the $Chisquare(k)$ distribution. Note that k is any positive value and is referred to as the *degrees of freedom*.

The values of the density curve for the $Chisquare(k)$ distribution can be obtained using the Calc I Probability Distributions I Chi-Square command, with k as the Degrees of freedom in the dialog box, or the session command **pdf** with the subcommand **chisquare.** For example, the command

```
MTB > pdf c1 c2;
SUBC> chisquare 4.
```

calculates the value of the $Chisquare(4)$ density curve at each value in C1 and stores these values in C2. This is useful for plotting the density curve. The Calc I Probability Distributions I Chi-Square command, or the session commands **cdf** and **invcdf,** can also be used to obtain values of the $Chisquare(k)$ cumulative distribution function and inverse distribution function, respectively. We use the Calc I Random Data I Chi-Square command, or the session command **random,** to obtain random samples from these distributions.

We will see applications of the chi-square distribution later in the book but we mention one here. In particular, if $x_1, \ldots, x_n$ is a sample from a $N(\mu, \sigma)$ distribution, then $(n-1)\, s^2/\sigma^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2 / \sigma^2$ is known to follow a $Chisquare(n-1)$ distribution, and this fact is used as a basis for inference about σ (confidence intervals and tests of significance). Because of the nonrobustness of these inferences to small deviations from normality, these inferences are not recommended.

6.6 Exercises

When the data for an exercise come from an exercise in IPS, the IPS exercise number is given in parentheses ( ). All computations in these exercises are to be carried out using Minitab, and the exercises are designed to ensure that you have a reasonable understanding of the Minitab material in this chapter. Generally, you should be using Minitab to do all the computations and plotting required for the problems in IPS.

If your version of Minitab places restrictions such that the value of the simulation sample size N requested in these problems is not feasible, then substitute a more appropriate value. Be aware, however, that the accuracy of your results is dependent on how large N is.

1. (6.6) Use the Stat I Basic Statistics I 1- Sample Z command to compute 90%, 95%, and 99% confidence intervals for μ.

2. (6.49) Use the Stat I Basic Statistics I 1- Sample Z command to test the null hypothesis against the appropriate alternative. Evaluate the power of the test with level $\alpha = .05$ at $\mu = 225$.

3. Simulate $N = 1000$ samples of size 5 from the $N(1, 2)$ distribution, and calculate the proportion of .90 z-confidence intervals for the mean that cover the true value $\mu = 1$.

4. Simulate $N = 1000$ samples of size 10 from the uniform distribution on (0,1), and calculate the proportion of .90 z-confidence intervals for the mean that cover the true value $\mu = .5$. Use $\sigma = 1/\sqrt{12}$.

5. Simulate $N = 1000$ samples of size 10 from the $Exponential(1)$ distribution (see Exercise II.4.7), and calculate the proportion of .95 z-confidence intervals for the mean that cover the true value $\mu = 1$. Use $\sigma = 1$.

6. The density curve for the $Student(1)$ distribution takes the form

$$\frac{1}{\pi} \frac{1}{1 + x^2}$$

for $-\infty < x < \infty$. This special case is called the *Cauchy* distribution. Plot this density curve in the range $(-20, 20)$ using an increment of .1. Simulate $N = 1000$ samples of size 5 from the $Student(1)$ distribution (see Exercise

II.4.12), and calculate the proportion of .90 confidence intervals for the mean, using the sample standard deviation for σ, that cover the value $\mu = 0$. It is possible to obtain very bad approximations in this example because the central limit theorem does not apply to this distribution. In fact, it does not have a mean.

7. Suppose we are testing $H_0 : \mu = 3$ versus $H_0 : \mu \neq 3$ when we are sampling from a $N(\mu, \sigma)$ distribution with $\sigma = 2.1$ and the sample size is $n = 20$. If we use the critical value $\alpha = .01$, determine the power of this test at $\mu = 4$.

8. Suppose we are testing $H_0 : \mu = 3$ versus $H_0 : \mu > 3$ when we are sampling from a $N(\mu, \sigma)$ distribution with $\sigma = 2.1$. If we use the critical value $\alpha = .01$, determine the minimum sample size so that the power of this test at $\mu = 4$ is .99.

9. The uniform distribution on the interval (a, b) has mean $\mu = (a + b)/2$ and standard deviation $\sigma = \sqrt{(b - a)^2 /12}$. Calculate the power at $\mu = 1$ of the two-sided z-test at level $\alpha = .95$ for testing $H_0 : \mu = 0$ when the sample size is $n = 10$, σ is the standard deviation of a uniform distribution on $(-10, 12)$, and we are sampling from a normal distribution.

10. Suppose that we are testing $H_0 : \mu = 0$ in a two-sided test based on a sample of 3. Approximate the power of the z-test at level $\alpha = .1$ at $\mu = 5$ when we are sampling from the distribution of $Y = 5 + W$, where W follows a *Student*(6) distribution (see Exercise II.4.12) and we use the sample standard deviation to estimate σ. Note that the mean of the distribution of Y is 5.